SYSTEM OF MECHANICS.

THEORY AND PRACTICE.

[illegible]

ORIGINAL AND PRACTICAL REMARKS

RULES, EXPERIMENTS, TABLES, AND CALCULATIONS

FOR THE USE OF PRACTICAL MEN

ILLUSTRATED BY SEVEN COPPERPLATE ENGRAVINGS

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PREFACE.

When so many excellent works are already written upon the science of Mechanics, it may seem needless, and even presumptuous, to offer any other work of the kind to public notice, unless it has something peculiar to distinguish it from its predecessors. It will therefore be necessary to acquaint the reader wherein this book differs from others.

There are some works now extant that combine theory and practice to a certain degree, but are so voluminous and expensive as to be beyond the reach of most artists, for whom they are chiefly intended; others, containing nothing but theory, involved in abstruse formulæ, which not one in ten thousand can either read or understand; while others, in the opposite extreme, treat of nothing

but detached, vague, and, in many cases, erroneous practical hints, without giving any reason whatever why a thing should be one way rather than another; and therefore (whatever might have been the good intentions of the authors) have no other tendency than to encourage ignorance, sloth, and laxity of inquiry, among such of our artists as may trust to them, and imagine they see clearly while groping their way in the dark.

In the following work I have endeavoured to combine theory with practice, and have condensed the useful and interesting matter of more voluminous publications into a small compass; by which any person of ordinary capacity may attain a knowledge of this science with little trouble. To understand the enunciations of the various propositions, and the practical rules and calculations deduced from them, the reader requires no farther previous attainment than a knowledge of decimal arithmetic. The demonstrations are concise, simple, generally original, and, I hope, sufficiently rigorous; most of them requiring only a knowledge of the rules of proportion, and none more than a slight acquirement in geometry or simple equations. Several propositions are brought into

one in many instances, and the same reading is applied to several figures at once. By such means' there are perhaps few books of this price wherein so much matter is scientifically demonstrated and explained.

In the application of the principles to practice, it cannot be expected, in a work of this size, that descriptions should be given of compound engines or machines; for they are now so numerous, that a volume is scarcely sufficient to contain a mere catalogue of them. Besides, some are so complicated and various, that a volume can scarcely describe one of them, particularly the steam-engine. But this work, I hope, contains the leading principles of all machines and structures, without a knowledge of which the artist would endeavour in vain to understand any of them. The particular subjects are enumerated in the table of contents.

I have bestowed much time and study upon this work; yet I fear the reader will discover errors and omissions that may have escaped my notice. Nothing that is the produce of human invention is perfect, much less this. I have treated this difficult subject according to the best of my weak ability; and that candid and ingenuous

readers may find in the following pages something useful, is the sincere wish of their very obedient servant,

THE AUTHOR.

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A

SYSTEM OF MECHANICS.

DEFINITIONS.

1. MECHANICS is a science that explains the nature, principles, and properties of bodies, either in a state of motion or rest, with respect to their actions upon one another.

2. When the science is applied to fluids at rest, it is called HYDROSTATICS; when to fluids in motion, HYDRAULICS.

3. Matter is any substance, the object of our senses.

4. A Body is any quantity of matter collected together.

5. Density of a body is its quantity of matter compared with its bulk.

6. Weight of a body is its quantity of matter without having regard to its bulk.

7. Motion is the act of a body while changing its place.

8. Absolute Motion is when the body actually changes its place.

9. Relative Motion is when the body is changing its place only with respect to other bodies. Thus, a person sitting in the cabin of a ship, while it is moving, has an absolute motion, but no relative motion with respect to the ship; and if he walks from stem to stern with the same velocity with which the ship moves forward, he has then no absolute motion, but has a relative motion with the ship.

10. Motion is said to be *uniform*, when it passes through equal spaces in equal times.

11. Motion is said to be *accelerated*, when it continually increases; and is said to be *retarded*, when it continually decreases.

12. The quantity of matter, multiplied by its velocity, is called the Quantity of Motion, or Momentum of the body.

13. The Velocity of a body is an affection of motion, by which it passes over a certain space in a given time, and is always measured by the space passed over in a given time.

14. Force is the cause of motion, and is always measured by the quantity of motion it produces in a given time. Force is said to be *constant* or *accelerative*, when it acts incessantly; *uniform*, when it produces equal effects in equal times; *variable*,

when the effects produced in equal times are unequal; and *impulsive*, when it acts by percussion, or in an imperceptibly small portion of time.

15. Gravity is that force by which all bodies endeavour to descend towards the centre of the earth.

16. Specific Gravity is the proportion of the quantity of matter contained in a body, compared with another body of the same bulk; which standard of comparison is commonly water.

17. Centre of Gravity of any body is a point in it, on which, if the body were suspended, it would rest in any position.

18. Equilibrium is when two or more opposite forces destroy one another's effects, and cause the body on which they act to remain at rest.

AXIOMS.

1. If a body be at rest it will remain at rest, and if in motion it will continue in motion uniformly in a straight line, if it be not disturbed by the action of some external cause.

2. The change of motion is always proportional to the force which causes it, and takes place in the direction in which the force acts.

3. The action and reaction of bodies upon one another are equal.

4. Heavy bodies always descend, or endeavour to descend, in straight lines perpendicular to the horizon, or towards the centre of the earth; and will descend to the lowest place they can get to.

Note. The second part of the first axiom is by no means self-evident, nor has it as yet been mathematically demonstrated: but it has been satisfactorily established by experience; for, in throwing a ball along a horizontal surface, the nearer we approach to perfection in doing away with friction and other obstructions, the longer will the ball move; and if the effects of the resistance of the air and of friction be fairly calculated, the result will exactly account for the body stopping.

In the following Treatise we are to suppose that all planes are perfectly even on the surface; all bodies perfectly smooth and homogeneous, and moving without friction or resistance; all lines perfectly straight and inflexible; all cords perfectly pliable. None of these suppositions are true; but proper allowance must be made for the defects when the theory is established.

LAWS OF MOTION.

PROPOSITION I.

The quantities of matter in all bodies are in the compound ratio of their magnitudes and densities.

PROPOSITION II.

The quantities of motion in all moving bodies are in the compound ratio of their quantities of matter and their velocities.

PROPOSITION III.

In uniform motions, the space passed over is in the compound ratio of the time and velocity.

PROPOSITION IV.

The motion generated by any impulsive force is in proportion to that force.

Demonstration. The truth of the above Propositions is evident from the Definitions and Axioms. From the three last we obtain the following general expressions:

Let $b =$ the quantity of matter of a body in motion.

$f =$ the impulsive force acting on b.

$m =$ the momentum, or quantity of motion in b.

Let v = the velocity generated in b.
s = the space described by the body b.
t = the time of describing the space s with the velocity v.

$$\text{Then } b : \frac{m}{v} : \frac{f}{v} : \frac{mt}{s} : \frac{ft}{s}$$

$$f : m : bv : \frac{bs}{t}$$

$$m : f : bv : \frac{bs}{t}$$

$$s : tv : \frac{tm}{b} : \frac{tf}{b}$$

$$v : \frac{m}{b} : \frac{s}{t} : \frac{f}{b}$$

$$t : \frac{s}{v} : \frac{sb}{m} : \frac{sb}{f}$$

PROPOSITION V.

No force can generate or destroy motion in a body instantaneously, or without time.

Demonstration. For there can be no motion but must comprehend space; but the motion is as the force that generates it, and the force is as the space multiplied by the quantity of matter divided by the time. Therefore, if the time be nothing, the force will be infinite, which is impossible.

Corollary. All forces are therefore accelerative; and any force acting on a body will accelerate its motion, either until the force ceases to act, or until

the body meet such a resistance as shall be equal to the force, and then it will move uniformly.

PROPOSITION VI.

All forces lose their effect, in proportion to the quantities of motion they generate in other bodies.

Demonstration. For all motions are caused by other motions: all forces, therefore, are but certain momenta of bodies acting on other bodies; and (Axiom 2,) the change or loss of motion is always proportional to the force which causes it, and in the same direction.

COMPOSITION AND RESOLUTION OF FORCES.

PROPOSITION VII.

If any body at A (fig. 1,) be acted on by two forces at the same time, one of which alone would move it uniformly over AB in a given time, while the other force alone would move it uniformly over AC at the same time. Complete the parallelogram ABDC. Then the united effects of the two forces would move the body along the diagonal AD, and it would arrive at D in the end of that time. This is called the Parallelogram of Forces.

Demonstration. For while the body moves uniformly along the line AC, let us suppose the line AC to be carried, with the body parallel to itself, uniformly along the parallelogram to BD. When the line has been carried to *ac*, the motion being uniform, the body must have moved through a proportional space, and will have arrived at *b*, a point in the diagonal; which is evident from the principles of similar triangles. The same may be shown for any other point that may be assumed, till it arrive at D.

Cor. The side BD being equal and parallel to AC, if the two forces be represented by the lengths and directions of the two sides of a triangle AB, BD, in succession, then the third side AD represents the equivalent force and direction of the two forces. This is called the Triangle of Forces.

PROPOSITION VIII.

If a body at A (fig. 2,) be acted on by three or more forces at the same time, one of which alone would move it uniformly over AB, while another in the same time would move it over AC, and another over AD; to find the equivalent force, and direction of all the forces.

From B draw BE equal and parallel to AC, and from E draw EF equal and parallel to AD; then AF represents the force and direction of the three forces, and F the point arrived at by the

body in that time. In the same manner the problem may be extended to any number of forces.

Demonstration. Because, by the last corollary, AE is the equivalent of AB, and AC (= BE), and AF is the equivalent of AE (= AB and AC), and AD (= EF). Therefore AF is the equivalent of all the forces.

PROPOSITION IX.

If three forces, acting upon a body at the same instant of time, be represented by three sides of a triangle AB, BD, and DA, (fig. 1,) and in the same directions taken in order, the body will remain at rest.

Demonstration. For it has been shown, that the force and direction represented by any one side is equivalent to the force and direction of the other two sides.

Cor. If three forces acting upon a body be in proportion to the three sides of a triangle drawn perpendicular to the three forces respectively, the body will also remain in equilibrio, because such a triangle will be similar to the former.

PROPOSITION X.

Any single force may be resolved into two or more forces. For since the forces represented by

AE and EF (fig. 2,) are equivalent to the force represented by AF, the force AF may be supposed to be resolved into the two forces AE and EF, and consequently into as many different pairs of forces as you can raise sides on the base AF: and each of these two forces again may be in the same manner resolved into any other two; so that the problem admits of an unlimited number of solutions.

PROPOSITION XI.

If any number of forces in different planes act upon a body, they may be reduced to other forces in the same plane. For if AB and BE (fig. 2,) be in a different plane from EF and FA, draw AE, which is equivalent to AB and BE, and AE is in the same plane with EF and FA.

PROPOSITION XII.

If three forces A, B, and D, (fig. 3,) in the same plane, keep one another in equilibrio, they are to one another in proportion to the sines of the angles through which their respective lines of direction do pass when produced.

Demonstration. Produce AC to E, and BC to F, and complete the parallelogram CFDE. Then the three forces are as DC, CF, and CE, or as the three sides of the triangle DCE, or as the sines of the opposite angles: that is, DE (or force B) is as the sine of DCE or DCA; likewise DC (or force

D) is as the sine of CED or ACF or ACB; and CE (or force A) is as the sine of CDE or DCF or DCB.

Cor. The above propositions hold equally true, whether the forces act by impulse, thrusting, or drawing, or whether by attraction or repulsion.

PROPOSITION XIII.

Any three forces that keep one another in equilibrio must tend to or from the same point as a centre.

Demonstration. Let the figure ABC (fig. 4, 5, 6,) be held in equilibrio by three forces in the direction A*a*, B*b*, and C*c*. Let A*a*, B*b*, meet when produced in the point O, the forces would act the same as if attached at O. Let the two forces be represented by the lines *a*O and *b*O, and compose them into the single force OD; this force would also tend to or from O. But the force OD is held in equilibrio by the force in direction C*c*; OD and C*c* must therefore be in the same straight line, and meet in O.

THE DESCENT OF BODIES IN FREE SPACE.

PROPOSITION XIV.

The space passed through by a body falling from rest in free space is in proportion to the square of the time of falling.

Cor. The time is as the square root of the space.

PROPOSITION XV.

The velocity acquired by a body falling from rest in free space is in proportion to the time of falling.

Cor. 1. The velocity acquired is in proportion to the square root of the space fallen through.

Cor. 2. The space fallen through is in proportion to the square of the velocity.

PROPOSITION XVI.

A body moving with the velocity acquired by falling through any height, will move through twice the space in the time of its fall.

Demonstration. Let the line 06 (fig. 7,) represent the time of a body falling from rest in free

space, and let the line be divided into a number of equal portions of time, 1, 2, 3, &c. Then, since the body, upon account of the constant action of gravity, must be uniformly accelerated, the spaces fallen through in equal portions of time must therefore be uniformly augmented.

This may be conveniently represented by the uniform augmentation of the distance of the two lines 06 and 0A, commencing at 0.

Draw the line 6A, (fig. 7,) and draw 1*a*, 2*b*, 3*c*, &c. parallel to 6A; also draw *ai*, *bp*, *cq*, &c. parallel to 06, and join 1*s*, 2*r*, 3*q*, &c. Then it is demonstrable that all the triangles within the figure are equal. Now the space that a body falls through in the first portion of time will be represented by the triangle 01; the space fallen through in the second portion of time, by the three triangles between 1 and 2; and the space fallen through in the third portion of time, by the five triangles between 2 and 3, and so on, augmenting uniformly by 2. Also, that the whole space fallen through in a given time is the sum of all the triangles above that time; which sum is evidently as the square of the time of falling. It further appears, that since the triangle 01*a* is the space that the body descends through in the first portion of time, that each of the triangles *ab*, *bc*, *cd*, &c. are the uniform augmentations of the spaces fallen through by the action of gravity alone; and therefore that all the

remainder of the spaces must have been descended through by the acquired velocity alone, which, when once acquired, must be continued, according to Axiom 1. From this it is evident that the spaces described by the acquired velocity at the end of the first portion of time, are the two triangles $1ag2$; and that, at the end of the second portion of time, the velocity is such as will cause the body to describe a space uniformly of eight triangles in the same time in which it described the first four, and so on.

Cor. 1. The same force is required to keep a body in any uniform motion upwards or downwards, as is required to keep it suspended or at rest.

Cor. 2. If a body be projected upwards with any velocity, the height of its ascent will be as the square of the time of ascending.

Cor. 3. If a body be projected upward with the velocity it acquired by falling in any time, it will in the same time lose all its motion, and will ascend to the same height.

Cor. 4. All the above propositions and corollaries hold equally true, of bodies descending or ascending upon inclined planes.

Let T = the time, V = the velocity, and S = the space fallen through;

$$\text{Then } V : T : \sqrt{S}$$
$$T : V : \sqrt{S}$$
$$S : T^2 : V^2.$$

It has been discovered by experiment, that a body falls through 16·095 feet in the first second of time. If we assume the time in seconds, and the space in one second = 16·095 feet, we obtain from the above propositions all the practical rules that may be required.

If T = the time in seconds, V = the velocity in feet per second, and S = the space fallen through;

Then $V = 32{\cdot}19T = \sqrt{64{\cdot}38S}$, $T = \frac{V}{32{\cdot}19} = \sqrt{\frac{S}{16{\cdot}095}}$, and $S = 16{\cdot}095T^2 = \frac{V^2}{64{\cdot}38}$.

Thus, if a body falls 100 feet, then, omitting the small fraction ·095, $\sqrt{64 \times 100} = 80$ = the velocity acquired in feet, and

$\sqrt{\frac{100}{16}} = \frac{10}{4} = 2\frac{1}{2}$ = the time in falling, in seconds.

Also, if a body falls $2\frac{1}{2}$ seconds, then

$32 \times 2\frac{1}{2} = 80$ = the velocity acquired in feet, and

$2\frac{1}{2}^2 \times 16 = 6\frac{1}{4} \times 16 = 100$, the height fallen through.

Again, if a body in falling acquire the velocity of 80 feet per second,

$\frac{80}{32} = 2\frac{1}{2}$ = the time in falling, and

$\frac{80^2}{64} = \frac{6400}{64} = 100$, the height fallen through.

THE MOTION OF PROJECTILES IN FREE SPACE.

PROPOSITION XVII.

If a body upon the surface of the earth be projected into free space, it will by its motion describe a parabola.

Demonstration. Let AD (fig. 8,) be the direction of its motion: then, if the body were without gravity, it would for ever move in the straight line AD (Axiom 1,) and describe equal spaces in equal times. But since gravity acts in lines perpendicular to the horizon, it does not affect the motion in direction AD, but generates a uniformly-accelerated motion towards the earth's centre. Let AFG, &c. be the curve the body describes, and let AB, BC, CD, &c. be all equal. Draw AM, BF, CG, DH, &c. perpendicular to the horizon, and complete the parallelograms AF, AG, AH, &c. Now in the time that the body would describe the spaces AB, AC, AD, &c. it will, by the force of gravity, descend through BF, CG, DH, &c., which are as the squares of the times in which they are described, (Prop. XIV.), that is, as the squares of the lines AB, AC, AD, &c. or KF, LG, MH, &c.; and BF, CG, DH, &c. are equal to AK, AL, AM, &c. Therefore the parts of the diameter of the

curve AK, AL, AM, &c. are respectively as the squares of the ordinates KF^2, LG^2, MH^2, &c., which is the property of the parabola. *Dr Simson's Conics*, Prop. XII. Cor. 1, Parabola.

PROPOSITION XVIII.

The horizontal distances of projections, made at any elevations, and with any velocities, are as the sines of double the angles of elevation, and the squares of the velocities jointly.

Demonstration. (Fig. 8.)

Let v = velocity of the projectile, or the space it describes in time 1.

d = descent of a body by gravity in time 1.

a = AE, the horizontal distance, or amplitude.

$\left.\begin{array}{l} s = \text{sine} \\ c = \text{cosine} \end{array}\right\}$ of twice the elevation.

In the right-angled triangle AEV, $c : a ::$ Rad. $1 : \frac{a}{c} =$ AV, and $c : a :: s : \frac{sa}{c} =$ VE, and v : time 1 :: (AV)$\frac{a}{c} : \frac{a}{cv}$ = time in describing AV (Prop. III.), and d : (time 1^2) $= 1 :: \frac{sa}{c}$(VE) $: \frac{sa}{cd}$ = square of the time of describing VE (Prop. XIV.) But the time of describing VE, AV, and the curve AGE, are all equal; wherefore $\frac{sa}{cd} = \frac{a^2}{v^2c^2}$ or $a = \frac{v^2sc}{d}$. But by trigonometry, $2sc$ is equal to the sine of twice

the elevation, which call S, radius being 1; whence a, the amplitude, is as Sv^2, $2d$ being a constant quantity.

Cor. The greatest random or horizontal projection is made with an elevation of 45 degrees; and the horizontal distances are equal at all elevations, equally distant above and below 45 degrees.

PROPOSITION XIX.

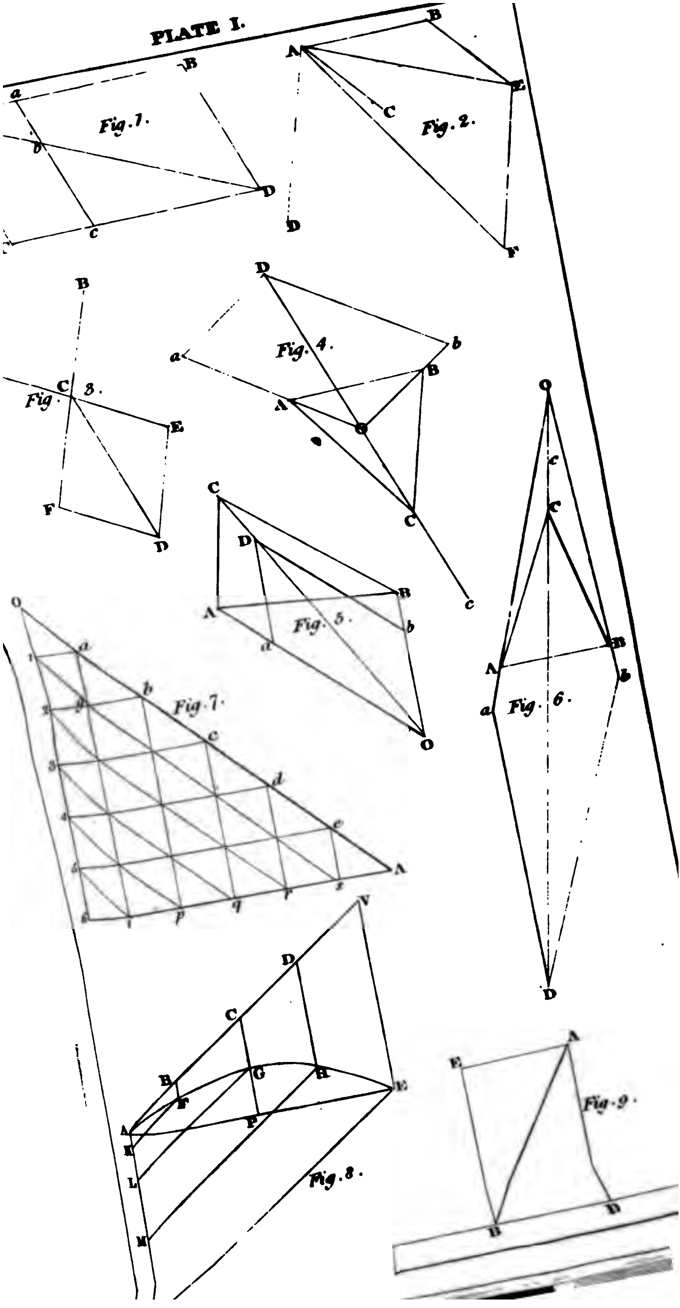
The altitudes of projections are as the squares of the sines of the elevations and the squares of the velocities jointly.

Demonstration. For if G (fig. 8,) be the greatest altitude of the curve, $CP = \frac{1}{2}VE$ (by sim. trian.), and $CG = \frac{1}{4}VE$ (Prop. XIV.); therefore $GP = \frac{1}{4}VE$. But $VE = \frac{sa}{c}$ (assuming the last notation). For a substitute its equal $\frac{v^2sc}{d}$; then $VE = \frac{v^2s^2}{d}$, or $GP = \frac{v^2s^2}{4d}$: wherefore GP is as v^2s^2.

From the above propositions we may derive the following equations:

Assume the above notation, and let t = time of flight in seconds, and h = the greatest altitude in feet;

PLATE I.



$$\text{Then } t = \frac{ac}{v} = \frac{vs}{d} = \sqrt{\frac{sa}{c}}$$

$$a = tcv = \frac{scv^2}{d} = \frac{t^2c}{s}$$

$$h = \frac{v^2s^2}{4d} = \frac{t^2d}{4}$$

$$v = \frac{a}{ct} = \frac{2\sqrt{hd}}{s} = \sqrt{\frac{ad}{sc}} = \frac{td}{s}$$

$$s = \frac{td}{v} = \frac{ad}{cv^2} = \frac{2\sqrt{hd}}{v} = \frac{t^2c}{a}.$$

General Remarks. What has been said of falling bodies, and the motion of projectiles, applies only when they fall or are projected in vacuo. If in air, or any other fluid, great allowance must be made for its resistance; for by reason of that the motion is continually retarded, and the balistic curve is made to deviate very considerably from a parabola. For farther information on this subject, see RESISTANCE OF FLUIDS.

INCLINED PLANE.

PROPOSITION XX.

IF one body acts against another body by any kind of force whatever, it exerts that force in direction perpendicular to the surface whereon it acts; and the effect of the force is in proportion to the sine of the angle of incidence.

Let the body B (fig. 9,) be acted on by the force AB in the direction AB; the force is exerted in direction EB, and its effect is in proportion to the length of EB, perpendicular to the surface of the body B.

Demonstration. Let the force AB be resolved into the two forces AD and DB, one perpendicular to the surface of the body B, and the other parallel to it; then the force DB can have no effect on the body B, and the whole effect must be by the force AD = EB perpendicular to it, and which force is as the sine of the angle of incidence ABD.

Cor. The same truth holds, whether the force act by impulse, pressing, or drawing; and whether constant or variable.

PROPOSITION XXI.

If a heavy body W (fig. 10, 11, 12, 13, 14,) be sustained upon an inclined plane AC, by a power P acting in any given direction WP, and if BD be drawn perpendicular to WP, to meet the side AC in D; then,

The weight of the body is in proportion to AB;
The power P, in proportion to DB; and
The pressure upon the plane in proportion to AD.

Demonstration. Because AB, DB, and AD, are respectively perpendicular to direction of gravity,

the power, and the pressure upon the plane; therefore (Prop. IX. Cor.) the body is in equilibrio, and the forces are as the three sides of the triangle ABD.

Cor. 1. The weight, power, and pressure upon the plane are respectively as the cosine of the angle of traction CWP, the sine of the plane's elevation CAB, and the cosine of the direction of the power above the horizon.

Cor. 2. If the line of traction be parallel to the plane, then the weight, the power, and the pressure upon the plane are respectively as the length of the plane AC, the height CB, and the base AB; or as radius, the sine, and cosine of the plane's elevation.

Cor. 3. If the line of traction be parallel to the horizon, (fig. 13,) then the weight, the power, and the pressure upon the plane are respectively as the base AB, the height CB, and the length AC; or as the cosine, sine, and radius of the plane's elevation. For in this case the point D falls in C.

Cor. 4. When the line of traction is parallel to the plane, the power is least: when the line of traction is perpendicular to the plane, the power and the pressure are infinite, because the lines CA and BD will never meet, being parallel; and when

perpendicular to the horizon, the power is equal to the weight, and the pressure is nothing, because the point D falls on A.

Cor. 5. If two bodies be in equilibrio upon two opposite inclined planes, their weights are as the lengths of the planes inversely.

EXAMPLE 1. What power is required to sustain a weight of 10,000 pounds upon an inclined plane rising 20 feet in 100 of its length, the line of traction being parallel to the plane? Also, what is the pressure upon the plane?

As the length 100 : height 20 :: 10000 : 2000 = the power.

$\sqrt{100^2 - 20^2} = 97{\cdot}98$ = length of base.

100 : 10000 :: 97·98 : 9798 = the pressure upon the plane.

EXAMPLE 2. If the friction of a loaded waggon be such as to sustain it upon an incline of 1 in 100, what power will it require to raise the above weight of such waggons on the above plane, as Example 1?

100 : 1 :: 9798 : 97·98 = the force of friction.

2000+97·98 = 2097·98 lb. = the power that will sustain it without motion: any thing greater will raise it.

Or assuming that a horse can raise 180 lb. through 60 yards in a minute; then 2097·98 ÷ 180 = 11·65 horse powers will raise the weight at the rate of 60 yards per minute.

PROPOSITION XXII.

The space which a body describes upon an inclined plane, descending from rest, is to the space which a body falling perpendicularly describes in the same time, as the height of the plane CB is to its length CA, or as the sine of the plane's elevation to radius.

Demonstration. (Fig. 15.) Because all that has been demonstrated respecting bodies falling perpendicularly, applies equally to bodies descending upon inclined planes, substituting the relative weight of the body, or the power that sustains it, in place of its absolute weight. Since action and reaction are equal, the force wherewith a body endeavours to descend upon an inclined plane is equal to the force that sustains it; and that is (Prop. XXI. Cor. 2,) to the force of gravity as CB to CA. But the spaces described are as the forces, that is, as CB to CA.

Cor. 1. If a body B (fig. 15,) falls perpendicularly from C, in the same time that a body E descends down the plane from C, the line joining E and B will always be perpendicular to AC.

Demonstration. Because the spaces described are as CA to CB; that is, as CB to CE.

Cor. 2. The time of a body descending down the plane CE (fig. 15,) is to the time of falling through the perpendicular height CD, as the length of the plane CE to its height CD.

Demonstration. Draw EB perpendicular to AC; then the time of descending through CE or CB : time of descending through CD :: $\surd$CB : $\surd$CD :: CE : CD.

Cor. 3. The velocity a body acquires by descending down the plane CA (fig. 15,) is equal to the velocity acquired by falling through the height CB.

Demonstration. For the force down the plane : CB :: force down the perpendicular : AC, and the time of descent down the plane : AC :: time down the perpendicular : CB. Therefore the force down the plane $\times$ time : CB $\times$ AC :: force down the perpendicular $\times$ time : CB $\times$ AC. Whence the force multiplied by the time in each are equal: but the force multiplied by the time is as the velocity; therefore the velocities are equal.

Cor. 4. A body will descend through any chord AB (fig. 16,) of a circle, in the same time that it

will fall perpendicularly through the diameter AC, if the chords meet either extremity of the diameter.

Demonstration. For if not, when the body descends down the diameter to C, let the other body be in D; ADC is a right angle (Cor. 1.). But ABC is also a right angle; D must therefore fall on B.

OSCILLATIONS OF PENDULUMS.

PROPOSITION XXIII.

THE times of descent of bodies through similar parts of similar curves are as the square roots of their lengths.

Demonstration. (Fig. 17.) For the curves may be conceived to be composed of an indefinite number of similar planes, of equal inclination where similarly situated. Therefore (Prop. XXII. Cor. 2,) the times of descent through AC, *ac* : time of a body falling through BC, *bc* :: AC, *ac* : BC, *bc*. But the time of a body falling through AC : time of falling through *ac* :: $\surd$AC : $\surd$*ac*. (Prop. XIV. Cor.).

Cor. 1. If pendulums describe similar arches, the times of their vibrations are as the square roots of their lengths.

Cor. 2. Their lengths are as the squares of the times of vibration.

Cor. 3. Their lengths are inversely as the squares of the number of vibrations in a given time.

Captain Kater found, by a set of very accurate experiments, the length of a pendulum that vibrates seconds to be 39·1386, in lat. 51° 31′ 8·4″, reduced to the level of the sea, in a cycloid, or in a very small arch of a circle.

Note.

Let T = the time of one vibration in seconds,
L = the length of the pendulum in inches,
N = the number of vibrations in one second;

$$\text{Then } T = \sqrt{\frac{L}{39{\cdot}1386}} = \frac{1}{N}$$

$$L = \overline{39{\cdot}1386T^2} = \frac{39{\cdot}1386}{N^2}$$

$$N = \sqrt{\frac{39{\cdot}1386}{L}} = \frac{1}{T}.$$

From the above general expressions, all the practical rules relative to the vibrations of pendulums can be obtained.

EXAMPLES.

1. If the length of a pendulum be 100 inches, then

$\sqrt{\frac{100}{39{\cdot}1386}} = \sqrt{2{\cdot}555} = 1{\cdot}598 =$ the time of one vibration, and

$\sqrt{\frac{39{\cdot}1386}{100}} = {\cdot}6257 =$ the number of vibrations in a second.

2. If the time of a vibration be 1·598, then
$1{\cdot}598^2 \times 39{\cdot}1386 = 2{\cdot}555 \times 39{\cdot}1386 = 100 =$ the length, and

$\frac{1}{1{\cdot}598} = {\cdot}6257 =$ the number of vibrations in a second.

3. If the number of vibrations in a second be ·6257, then

$\frac{1}{{\cdot}6257} = 1{\cdot}598 =$ the time of one vibration, and

$\frac{39{\cdot}1386}{{\cdot}6257^2} = 100 =$ the length of the pendulum.

General Remarks. From the above rules it may be deduced, that if twice the number of threads in 1 inch of the screw of a pendulum be multiplied by the time in minutes that a clock gains or loses in 24 hours, and the product divided by 37, the quotient will be the number of threads that the bob must be screwed up or down, to make it beat seconds.

A pendulum vibrating in a circular arc, of the same length as another pendulum vibrating in a cycloid, or in an infinitely small arc, loses a number

of seconds in 24 hours, equal to $\frac{5}{3}$, the square of the number of degrees of the arc the pendulum describes on each side of the vertical; and consequently, if a seconds pendulum goes right, by altering the length of the arc more or less; then $\frac{5}{3}$ the difference of the squares of the degrees described before and after the alteration, will give the number of seconds lost or gained in 24 hours.

Thus, if a clock goes right, with a pendulum vibrating arcs of 3° on each side of the vertical, and be altered to 6°; then $\frac{\overline{6^2-3^2}\times 5}{3}=\frac{\overline{36-9}\times 5}{3}=\frac{125}{3}=41\frac{2}{3}$, the seconds lost in 24 hours.

It may be shown, from the principles of the cycloid, (a subject too complicated for the nature of this work,) that, as the circumference of a circle is to its diameter, so is the time of one vibration to the time in which a body would fall through half the length of the pendulum. Therefore $3{\cdot}141593 : 1 :: 1^s : \frac{1^s}{3{\cdot}141593}$, and $\frac{1}{3{\cdot}141593} : \frac{39{\cdot}1386}{2} :: 1 : 193{\cdot}15$ inches, or $16{\cdot}095$ feet, the space a body falls through in one second.

THE MECHANIC POWERS.

The Mechanic Powers are, 1. The Lever; 2. The Wheel and Axle; 3. The Pulley; 4. The Screw; 5. The Wedge; and by some the Inclined Plane, which has already been explained.

Note. The body to be moved is called the *Weight*, and the force that moves it is called the *Power*.

THE LEVER.

A Lever is an inflexible beam or bar of wood or metal, supported on a certain point called its Fulcrum. There are three kinds of levers.

1. When the fulcrum is between the weight and the power. (Fig. 18.)

2. When the weight is between the fulcrum and the power. (Fig. 19.)

3. When the power is between the weight and the fulcrum. (Fig. 20.)

Any of these levers may be either straight or bended.

PROPOSITION XXIV.

Any lever will rest in equilibrio over its fulcrum, if the weight W (fig. 18, 19, 20,) be to the power P as the distance between the power and fulcrum PF to the distance between the weight and fulcrum WF.

Demonstration. Let AB (fig. 21,) represent a homogeneous cylinder: if laid over a fulcrum, it would evidently be supported in the middle point O, the same as if the whole mass were concentrated in that point. But the cylinder may be conceived to be divided into two unequal portions, AC and BC, which would be separately sustained at their middle points D and E; wherefore the weight of AC acting at D, and the weight of CB acting at E, would balance the inflexible line DE, if upheld at the centre O; which point would sustain the whole weight of AB. But $OD = AO - AD = \frac{1}{2}AB - \frac{1}{2}AC = \frac{1}{2}BC$, and $OE = OB - EB = \frac{1}{2}AB - \frac{1}{2}CB = \frac{1}{2}AC$; consequently $OD : OE :: BC : AC ::$ weight concentrated in E : weight concentrated in D. This is true in the other two orders of levers also; for if W (fig. 19, 20,) were removed to an equal distance beyond F, they would become levers of the first order, and the direction

of the force would be reversed in P, while the action on P would remain equal in quantity.

Otherwise thus:—Because the weight, power, and force upon the fulcrum all tend to or from the earth's centre, which let be C, (fig. 41, 42, 43.) But the three forces are to one another as the sines of the angle through which they pass (Prop. XII.); that is, the force F, in direction FC, is as the sine PCW; the force P as the sine WCF; and the force W as the sine PCF. But because the three angles are exceedingly small, their sines are respectively as PW, FW, and PF; therefore the force P is as FW, the force W as PF, and the force on the fulcrum F as PW.

PROPOSITION XXV.

Levers of any form, whether crooked or straight, will be in equilibrio, in whatever directions the power and weight act, if the weight W be to the power P inversely as the perpendiculars let fall from the fulcrum upon their several lines of direction (fig. 22, 23, 24, 25,); or they will be in equilibrio when $W : P :: PF \times \sin. pPF : WF \times \sin. wWF$.

Demonstration. Because the lever pFw (fig. 22, 23, 24, 25,) would be in equal equilibrio, and the power at P is evidently the same as that at p, which may be supposed to be the continuation of a

stretched rope, and the same may be said of the weight at W and *w*.

Cor. 1. The power has the greatest effect when it acts perpendicularly to the direction of the lever.

Cor. 2. If a lever WFP (fig. 26,) be fixed to an axis F*f*, it will be in equilibrio when the power is to the weight inversely, as the perpendiculars let fall upon the axis F*f* from the power and the weight, or when P : W :: WF : P*f*.

Cor. 3. (Fig. 18, 19, 20.) When a straight lever is in equilibrio, and the weight and power acting in parallel directions; the power, the weight, and the pressure upon the fulcrum, are each in proportion to the distance of the other two, or W is in proportion to PF, P in proportion to WF, and the pressure upon the fulcrum in proportion to WP.

Cor. 4. (Fig. 27.) When several weights are suspended on a lever, if the sum of all the products of each weight, by its distance from the fulcrum on one side, be equal to the sum of all the products on the other side, then the lever will be in equilibrio.

The Balance is a species of lever, having the fulcrum in the centre. The properties of a good balance are, 1*st*, That the points of suspension of the scales, and the centre of motion, or fulcrum of

the beam, be all in one straight line. *2d*, That the arms of the beam be of equal length from the fulcrum. *3d*, That they be as long as possible with convenience. *4th*, That there be as little friction as possible. *5th*, That the centre of gravity of the beam be very little below the centre of motion. *6th*, That they be in equilibrio when empty. A good method of proving a balance is to reverse the weights from one scale to the other; if they are still in equilibrio the balance is just, if not it is false. If a commodity be weighed in a false balance, first in one scale, and then in the other, a mean proportional between the two weights is the true weight of the article.

Such instruments as smiths' tongs, pinches, nippers, scissors, a ship's helm, &c. are levers of the first order.

Handspikes, wheel-spokes, a wheel-barrow, &c. are levers of the second order.

The bones of animals, tongs, &c. are levers of the third order.

THE WHEEL AND AXLE.

PROPOSITION XXVI.

In the wheel and axle, if the power P (fig. 28,) be to the weight W as the diameter of the axle to the diameter of the wheel where they respectively

act, the power and the weight will be held in equilibrio.

Demonstration. For the centre of motion F is the fulcrum of a lever, whose arms are wF and pF, the radii of the axle and wheel.

Cor. (Fig. 29.) When the power is not a tangent to the diameter, but acts in direction PP, the weight and power are in proportion to pF and wF.

Demonstration. Because the lever is wFP.

Instead of a wheel, spokes fixed into the axle, of the same length as the radius of the wheel, and any kind of force applied, instead of a weight, has precisely the same effect.

Such machines are the gin, the crab, the windlass, the capstan, &c. The water in most of the buckets of an over-shot wheel acts by its weight, according to the principle described in the corollary. (Fig. 29.)

PROPOSITION XXVII.

To calculate the power of any combination of wheels, with teeth acting into one another.

(Fig. 30.) As the product of the number of teeth or leaves of each pinion or leader is to the product of the number of teeth of each wheel or follower so is the power to the weight. If the first

pinion A be driven by a crank or lever BC, the ratio of the power to the weight will be augmented or diminished in proportion as the lever is longer or shorter than the radius of the axle FG which winds up the weight.

(Fig. 30.) If in the combination of wheels in fig. 30, the pinion A has 10 leaves, and the pinion E 8, their product is 80; and, if the wheel D has 60 teeth, and F 50, their product is 3000; so that the power is to the weight as 80 to 3000; but this power is modified by the length of the lever BC, and the radius of the axle FG. Let the lever be 15 inches, and the radius of the axle 6 inches, then $80 \times 6 = 480$, and $3000 \times 15 = 45000$, and 45000 divided by $480 = 93\frac{3}{4}$; so that the weight is to the power as $93\frac{3}{4}$ to 1; and if a man work at the winch with a power of 30lb., then $93\frac{3}{4} \times 30 =$ $2812\frac{1}{2}$ lb., = the weight he can raise, deducting allowance for friction from this.

Demonstration. The truth of this is easily deduced from the principles of the lever; for the number of the teeth of wheels acting into one another ought to be in proportion to the lengths of their radii.

The strength of the spindle A should be to that of ED as the radius of the pinion A to the radius of the wheel D; and the strength of the spindle ED to the strength of the axle FG : : radius of the pinion E : radius of the wheel F. Also the strength

of the pinion and wheel A and D should be to the strength of the pinion and wheel E and F as radius of E to the radius of D, otherwise it is evident that the strength is not proportional to the strain.

Note. The teeth of wheels ought to be epicycloids; but, as the subject is too long for an account of them, see Camus, or Dr Brewster's Appendix to Ferguson's Lectures.

PROPOSITION XXXVIII.

What has been said concerning wheels with teeth, is equally applicable to wheels driven with ropes or straps passing over them; for in the calculations it is but substituting the radius or diameter of the wheels or sheaves for the number of teeth.

In wheels driven with ropes, the ropes should run in grooves upon the circumference of the wheels; but, if they are driven with straps, the circumference of the wheels ought to be convex, as in fig. 31, otherwise it will not remain upon the wheel; the belt always climbs to the greatest circumference.

Demonstration. If the belt were put on near the edge of the wheel, as in fig. 32, then, by reason of its elasticity, the one side being more stretched than the other, it would have a flexure towards the other side of the wheel, and, as the wheel revolves, would come into contact with successive points of

PLATE II.

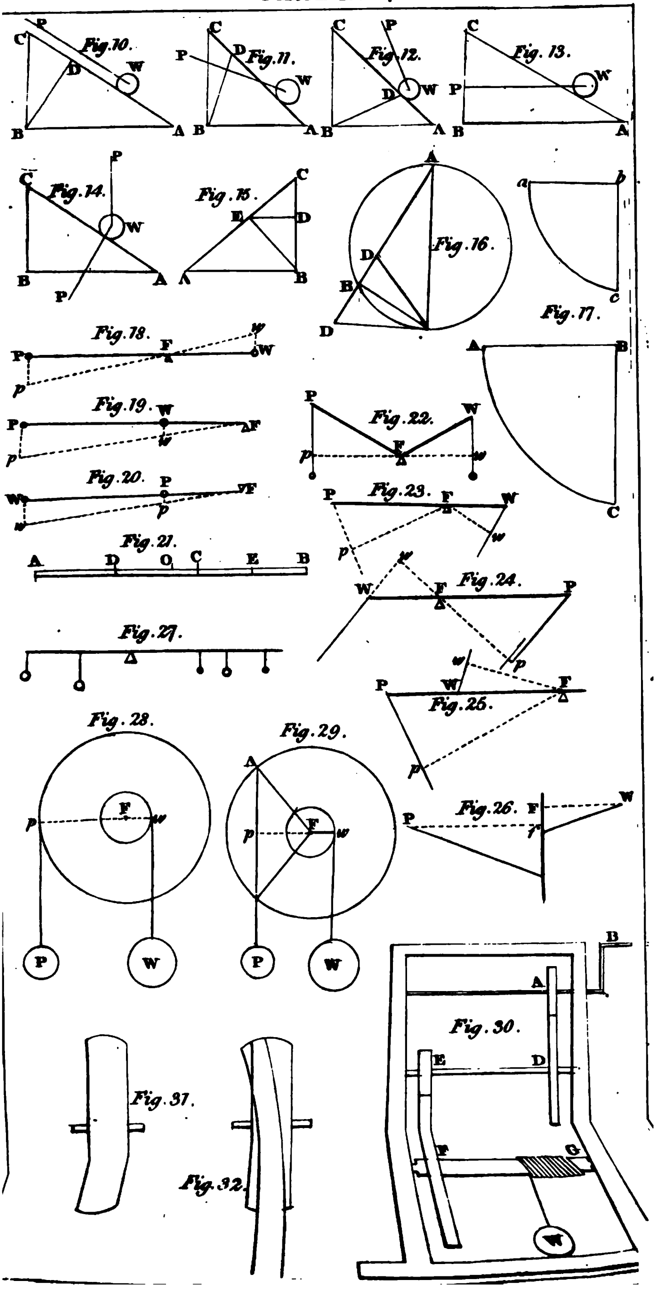


Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23.
Fig. 24.
Fig. 25.
Fig. 26.
Fig. 27.
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
Fig. 32.

the wheel still nearer the other side, until both edges of the belt were equally stretched, when the flexure would cease, and incline to neither side.

OF THE PULLEY.

PROPOSITION XXVIII.

(Fig. 33.) If a weight sustain a power by means of a rope passing over a fixed pulley, the power is equal to the weight.

Demonstration. Because all the parts of the running rope are equally stretched, and the centre of the sheave is the fulcrum of a lever, whose arms are the radii of the sheave, and therefore equal.

Cor. 1. The same holds with a rope passing over any number of fixed pulleys.

Cor. 2. If a power P (fig. 34,) sustain a weight W by means of a rope passing over a moveable pulley, with one end of the rope fixed, the power is to the weight as 1 to 2.

Demonstration. For the rope at *a* and *b* is equally stretched, therefore each part bears half the weight.

Cor. 3. If a power P (fig. 36,) sustain a weight W by one running rope in any combination of pulleys, the power is to the weight as 1 to the number of parts of the rope attached to the moveable block A.

Demonstration. For every part of the same running rope must bear the same part of the weight, being all equally stretched.

Combinations of this kind are more convenient when all the wheels in each block move upon the same axle, passing through the block. It is shown, as in fig. 36, for the purpose of explanation.

Cor. 4. The combinations of pulleys in fig. 37, 38, 39, have a number of moveable pulleys attached to separate ropes, whereon are marked the part of the weight that each part of the ropes bears, deduced from the circumstance, that every part of the same running rope is equally stretched, and bears an equal part of the weight.

Many other combinations might be shown; but all combinations of pulleys, with a number of blocks, though powerful, have the disadvantage of moving the weight but a very little way, till the blocks are choked against one another, and therefore very inconvenient in practice.

In all combinations of pulleys, each rope and block should have strength in proportion to the part of the weight it has to sustain.

OF THE SCREW.

PROPOSITION XXIX.

THE power of a screw : is to the weight it will raise, or the resistance it will overcome : : as the distance between the centres of two threads of the screw : to the space passed over by the power in the same time that the screw moves between the two threads.

Demonstration. Because the motion of a screw may be reduced to the motion of a body upon an inclined plane, combined with a lever. If the lever were in length just the radius of the screw, then the power : would be to the weight : : as the distance between two threads : to the circumference of the screw; but when the lever is longer, as is always the case, the power must be to the weight in the above ratios, multiplied by the length of the lever, which is as the spaces passed over.

Thus, if a screw has 8 threads in an inch, and is moved by a lever, the end of which describes a circumference of 72 inches for one turn of the screw, then the power : is to the weight : as $\frac{1}{8}$: to 72, or as 1 : to 576.

Cor. 1. The same holds true in the double or

triple screw, by taking the distance of two threads of the same spiral; and the same is true also in the perpetual screw.

Cor. 2. If the screw be connected in any manner with the other mechanic powers, the power of the screw may either be augmented or diminished by the operation of the powers so connected.

A very great allowance must be made in estimating the power of the screw, on account of friction, for we find in ordinary cases the friction to be greater than the whole weight; for the friction is capable of supporting the whole weight when the power is wholly removed.

(Fig. 98.) Is a new perpetual screw, invented by Mr George Blackie, clockmaker, Musselburgh, applied to the circumference of the great circle of his superb engine, for graduating circles, and cutting the teeth of clock wheels.

The teeth of the wheel, as also the spiral of the screw, are equilateral triangles. The centre of the spiral is the only point having the base of the thread parallel to the axis of the screw; every other point is successively deflected from that direct position, by a quantity equal to the deflection of the arc from the axis of the screw; and by this means there is a perfect adaptation of the spiral to the teeth of the wheel.

The advantages of this screw above the common one are great and evident. The common screw

can act but upon one tooth at a time, and that, too, by single and successive points of the spiral; unless therefore every part be perfect, (which is impossible,) its motions and operations must be very irregular; and, admitting that it approximates to perfection in its original construction, yet by wearing it must soon lose that degree of perfection.

But in the construction of the screw here presented, the spiral acts upon a large portion of the circumference, and must consequently afford a compensation for errors, with a degree of accuracy proportioned to the number of teeth on which it acts: it has also this obvious and valuable advantage, above all others, that the longer it works, and the more it is worn, it becomes the more accurate in its operations, owing to the more perfect adaptation of the teeth to the spiral.

OF THE WEDGE.

PROPOSITION XXX.

If a wedge (fig. 40,) be in the form of an isosceles triangle, and its inclined sides parallelograms, then the power of the wedge : is to the whole resistance :: as the back of the wedge : to the sum of the two inclined sides.

Demonstration. Because the resistance acts in

direction AD and BD, perpendicular to the sides, (Prop. XX,) and the power in direction DE; they are therefore in equilibrio, (Prop. IX. Cor.)

The power of the wedge is much modified by the nature of the various substances on which it acts; so much so, that it is impossible to give any general rule for its operations in all cases.

PROPOSITION XXXI.

By the action of all mechanical agents, whatever is gained in power is lost in velocity; and whatever is gained in velocity is lost in power; or the product of the power into the space it passes over in any time, is always equal to the product of the weight into the space it passes over in the same time; or, the momentum of the effect is always equal to the momentum of the cause.

Demonstration. Because in all the orders of levers, (fig. 18, 19, 20,) let the power move the weight through the space Ww, then the power will have moved through Pp.

But PF : WF : : W : P, by the lever.
PF : WF : : Pp : Ww, by similar triangles
Pp : Ww : : W : P,
Or, $Pp \times P = Ww \times W$.

What has been demonstrated of levers, is evidently true in the wheel and axle; and can b

traced through all the combinations of wheels acting upon one another. Again in the pulley, (fig. 33,) when the weight and power are equal, it is evident that, when the pulleys move, their velocities will be equal, and in the pulley, (fig. 34,) when the weight is moved through any space, the power will have moved through twice that space; for, (fig. 35, which is but a modification of fig. 34,) when the weight has ascended or descended through any space, the parts of the rope *a* and *b* will both be shortened, or lengthened by that space: therefore the power P must have descended, or ascended, through both these spaces. The same law may be traced through any combination of pulleys, or any combination of mechanic powers whatever.

Cor. 1. Wherefore, if we know the velocity of the first mover, and the last of any machine, we also know the ratio of the power to the weight, and the reverse.

Cor. 2. All the advantage we can derive from powerful mechanic powers, or their combinations, is to raise ponderous weights, or to overcome such powerful resistances as cannot be effected otherwise; or to generate such rapid velocities, as grinding-wheels, turning-lathes, millstones, spinning machinery, &c. or for regulating motion as in clock work.

PROBLEM.

When two shafts are connected by a number of wheels and pinions between them, and the ratios of the velocities of the two shafts given; to find the number of teeth upon the connecting wheels and pinions.

The number of the cotemporary turns of a wheel and pinion acting into one another are reciprocally proportional to the numbers of their teeth.

EXAMPLE.

If the pinion *b* (fig. A, pl. VI.,) has 6 leaves, how many teeth must the wheel A have, to turn the pinion 8 times for the wheel's once?

1 : 8 : : 6 : 48 number of teeth.

When wheels and pinions interpose between them. Multiply the number of the leaves of all the pinions together, and that product by the number of turns the last must have for one of the first; decompose the product into all its factors, divide those factors into a number of bands equal to the number of interposing wheels, and the product of each band will be the number of teeth of the wheels.

EXAMPLE I.

If the pinion C is to turn 60 times for 1 turn of the wheel A, and the pinions *b* and *c* have each 6 leaves; what must be the number of teeth of the wheels A and B?

$60 \times 6 \times 6$, decomposed into all its factors, (which being multiplied together, will produce the same number,) gives 2, 2, 3, 5, 2, 3, 2, 3; divide these factors into two bands at pleasure, and we obtain many varieties of the numbers of teeth that the wheels A and B may have, namely,

$$\text{1st, } \begin{cases} 2 \times 2 \times 3 \times 5 = 60 = \text{A or B.} \\ 2 \times 3 \times 2 \times 3 = 36 = \text{B or A.} \end{cases}$$

$$\text{2d, } \begin{cases} 3 \times 3 \times 5 \qquad\qquad = 45 = \text{A or B.} \\ 3 \times 2 \times 2 \times 2 \times 2 = 48 = \text{B or A.} \end{cases}$$

Many other pairs may be found, the most convenient of which may be selected.

EXAMPLE II.

If the pinion *e* turns 720 times for 1 turn of the wheel A, and the pinions *b*, *c*, *d*, and *e* have each 6 leaves; required the number of teeth that the interposing wheels A, B, C, and D, must have.

$6 \times 6 \times 6 \times 6 \times 720$, decomposed into all its factors, gives 3, 2, 3, 2, 3, 2, 3, 2, 2, 2, 3, 5, 2, 2, 3; divide those factors into 4 bands at pleasure, namely,

$$\text{1. } \begin{cases} 2 \times 3 \times 5 \qquad = 30 \\ 2 \times 2 \times 2 \times 3 = 24 \\ 2 \times 2 \times 3 \times 3 = 36 \\ 2 \times 2 \times 3 \times 3 = 36 \end{cases} \qquad \text{2. } \begin{cases} 3 \times 3 \times 5 \qquad\qquad = 45 \\ 3 \times 2 \times 2 \times 2 \times 2 = 48 \\ 3 \times 3 \times 2 \qquad\qquad = 18 \\ 3 \times 2 \times 2 \times 2 \qquad = 24 \end{cases}$$

In the same manner many other sets may be found.

THE PRINCIPLE OF BEVEL GEAR.

When one wheel AB, is inclined to another BD, in any given angle whatever, (fig. B, C, pl. VI.) having their diameters given, which must be in proportion to the numbers of their teeth; to find their respective bevels.

Draw the diameter AB, and from its extremity B, draw BD in its given position; bisect the diameters in E and F; draw EC and FC perpendicular to AB and BD: the point C is the common vertex of two cones, to which the bevels and lines of the teeth must all concur.

PRACTICAL REMARKS ON THE FIRST MOVERS OF MACHINERY.

The forces generally employed, as the first movers of machinery, are, animal strength, water, wind, and that most noble instrument of power the steam engine, which last, to give an account of, with all its modifications, would require a large volume,

The human body is peculiarly adapted for the first mover of machinery, on account of the ready facility and intelligence by which it can apply its strength. An ordinary man can turn a winch with the force of 30 lb. for 10 hours a day, with the velocity of 2½ feet per second. But 2 men working at a windlass, with the handles at right angles to one another, can raise a weight of 70 lb. as easy as one man singly can raise 30 lb. This is owing to the force of the one always being greatest, in the position where the other is least. I might here trace the strength of the human body, in all its various applications; but this is already generally known.

A good draught horse (according to Messrs Watt and Boulton) can raise a weight of 32,000 lb. perpendicularly through 1 foot in a minute, or 180 lb. at the rate of little more than two miles an hour.

For the force of wind and water, see *Resistance of Fluids* at the end of this volume; and for the steam engine, see the many excellent works already written upon that subject.

It has been discovered, that the quantity of work done by any working animal is greatest in a given time, when the animal moves with one-third the velocity with which it is able only to move itself, and is loaded with four-ninths of the load it is able only to move.

The strength of men and all animals is greatest

when applied to a resistance at rest: when this resistance is overcome, and the animal in motion, its strength is diminished, until with a certain velocity it can only keep up the motion of its own body.

The last remark is not only applicable to the force of animals; but also to all moving forces in nature, in so far as their strength is diminished in proportion to the motion they convey to other bodies.

Thus, water or wind has the greatest power against the floats or sails of a mill when the mill stands still, and decreases as the velocity of the mill increases; and were it possible for the wheel to move with the velocity of the water or wind, it is evident they could have no force whatever upon the wheel; for it is the relative velocity only that can affect the wheel. The same may be shown of the wind upon the sails of a ship. The same law evidently regulates the power of a steam engine.

All these agents, however, can have no effect when the body to be moved stands still; neither can they have any effect when the velocity of the body moved is equal to the velocity of the agent. The body must therefore move with some intermediate velocity between these two extremes, to have the greatest possible effect; and that velocity can be shown to be one-third the velocity the moving agent would have when meeting with no resistance.

Gravity, or terrestrial attraction, seems on a su-

perficial view of the subject, to militate against this general law, as certainly it does, as far as regards all bodies on or near the earth's surface. This is owing to the immense quantity of matter in the earth, compared with the small portions that come under our immediate observation. Gravity, however, like all other agents, must move bodies in proportion to the magnitude of its power: that is so great, that the magnitudes of all bodies subject to our operations vanish before it.

Gravity is therefore the only force in nature that can accelerate motion, and remain itself constant and invariable.

FRICTION.

Friction is the force of obstruction to motion, which arises from one surface rubbing upon another: it is the only force in nature that is perfectly inert; it destroys motion and generates none. Friction is a main cause of the stability in the structure of machinery and buildings, and is necessary to the exertion of the force of animals; without it nothing could have stability, but in the lowest situation.

The force of friction varies according to the different substances employed, and can be ascertained only by accurate experiments made with the differ-

ent substances. The friction of planed woods, and metals polished without grease on one another, has been ascertained by experiment to be about one-fourth of the pressure, or that they will slide down a plane elevated about 14 degrees: That the friction does not increase with the increase of the rubbing surfaces: That, when the surfaces of wood, and other soft substances, are some time in contact without motion, the friction is greater than when after they begin to move; but not so with metals, and other hard substances: And, lastly, that friction does not increase with the increase of velocity; but is equal whether the motion be quick or slow, except in soft or heterogeneous substances, where it does increase a little, as might be expected; for in this case it partakes partly of the resistance of the bodies.

A cylinder of elm, 1 inch in diameter, will roll down a plane surface of oak, elevated 4 degrees; but if the cylinder be 4 inches in diameter, it will roll down the same surface elevated 1 degree.

The loss of force in rolling is therefore inversely as the diameter of the cylinder.

From the circumstance of the friction of hard substances being equal in all velocities, it is the only force that can be compared with gravity, which is also equal in all velocities. Friction may therefore be compared to the ascent of a body upon an inclined plane, moving without friction, or

to the force that will sustain a body upon an inclined plane.

From the above circumstance alone, we can discover the friction of wheel-carriages, and consequently the strength exerted by draught animals, without any machine for that purpose.

Thus, if on a dead level railway we impel a loaded waggon, whose weight is known, uniformly over a given measured distance per minute, and then withdraw the impelling force, the waggon will stop at a certain distance, which can also be measured, and the problem is reduced to the following:

Given the length of an inclined plane, and the velocity a body acquired by descending it; to find the force that will sustain a given weight upon that plane, which force is the friction of the waggon.

EXAMPLE.

If a waggon, weighing in all 4500 lb., be impelled at the rate of 360 feet in a minute, uniformly upon a dead level railway, and, after withdrawing the impelling force, the waggon stops at the distance of 74 feet 8 inches. Required its friction.

$\frac{360}{60} = 6$ feet $=$ the velocity per second.

$\frac{6^2}{64\frac{1}{3}} = \frac{36}{64\frac{1}{3}} = \cdot 56 =$ the height of the plane. (Prop. XVI. Cor. 4, and Prop. XXII. Cor. 3.)

$74\frac{2}{3}$ feet : ·56 :: 4500 lb. : 33·75 lb. = the sustain-

ing force, or the friction of the waggon. (Prop. XXI. Cor. 2.)

Or, 4500 : 33·75 : : 100 : ·75, or $\frac{3}{4}$ lb. for every 100 drawn.

If a horse draw 6 such waggons, in this case the force he exerts is $\frac{4500 \times 6 \times \cdot 75}{100} = 202\frac{1}{2}$ lb. with any velocity.

The friction being found as above, let it be required to determine the incline of a railway, such that the waggons being started with any velocity, would move on uniformly with that velocity; then it is evident that the incline in the above case must be 9 inches in the 100 feet; because the force of friction and the force down the plane are equal.

Let there be given the weight of a loaded waggon; the time in which it descends down a given space by gravity alone, upon a given inclined plane; to find its friction.

Rule. Find the time in seconds in which it would descend without friction, (Prop. XVI. Cor. 4, and Prop. XXII. Cor. 2.;) also find the force that would sustain it upon the plane, (Prop. XXI. Cor. 2.) Multiply the two numbers together, and divide their product by the time in seconds in which the waggon really descends. Subtract the quotient from the force that sustains it upon the plane as found above, the remainder is the friction.

EXAMPLE.

If a loaded waggon, weighing 4500 lb., descend down 120 feet of an incline of 1 foot in 100, in 60 seconds, what is its friction?

$100 : 120 :: 1 : 1{\cdot}2$ the height of the plane.

$\sqrt{\frac{1{\cdot}2}{16\frac{1}{12}}} \times 120 = .2731 \times 120 = 32{\cdot}772$ seconds = time of descent without friction. Again, $100 : 1 :: 4500 : 45 =$ force that sustains it without friction.

$45 - \frac{45 \times 32{\cdot}772}{60} = 45 - 24{\cdot}579 = 20{\cdot}421$ lb. = the friction.

Investigation of the Rule.

Let $a =$ the force that sustains the waggon without friction;

$b =$ the time of descent without friction;

$c =$ the time of the actual descent; and

$x =$ the friction; then $a - x$ is the true force down the plane; but the times are inversely as the forces; therefore $a : a - x :: c : b$, or $ac - ab = cx$, or $x = a - \frac{ab}{c}$.

DESCRIPTION

OF AN

INSTRUMENT FOR MEASURING THE FRICTION OF WHEEL-CARRIAGES.

INVENTED BY THE AUTHOR.

FIG. 45 is a machine for this purpose, fig. 46 is an end view of the same. AB is a frame of wood or iron. C is a hollow cylinder, moving upon a fixed axle, within which is inserted a spiral spring, on the same principle as the main-spring of a watch, but much stronger. D is another cylinder, fixed to the former, and moving on the same axle, (out of sight in fig. 45,) round which a rope R is coiled. On the edge of the cylinder D is a pinion with teeth, (not shown in the plan,) acting upon the wheel E, whose spindle moves the index F, which points out the quantity of resistance upon a dial-plate. Between the two cylinders C and D, is a wheel acting upon the pinion G, on whose spindle is the fly-wheel H.

CONSTRUCTION AND ADJUSTMENT OF THE MACHINE.

Having obtained the spring fitted into the cylinder C, and fixed upon the axle; fix it into the frame, by coiling a cord round the cylinder C; try what weight it can bear without damage, also how many turns the cylinder makes in doing so.

Then, as the force you wish to sustain : is to the force it actually sustains :: so is the diameter of the cylinder C : to that of D. And as 1 : is to the number of turns of the cylinder :: so is the diameter of the pinion D : to that of the wheel E. The size of the pinion G, depends upon the weight and diameter of the fly-wheel, and is best discovered by trials.

Thus, if we find that the spring bears about 133 lb., and we want it to bear 400, and if the diameter of C be 15 inches; then 400 : 133 :: 15 : 5 inches the diameter of D. And if the cylinder makes 4 turns; then as 1 : 4 :: 10 (say the number of teeth in D) : to 40 = the number of teeth in E, so that the index may go but once round.

Now coil the rope round the cylinder D, and lay it over a fixed pulley P, the machine being fixed at the same time; let the spring be perfectly relaxed, without any weight; turn the point of the index F, to the top of the dial-plate, and there mark the beginning of the divisions. Hang 40 lb. upon the

rope at W, and mark the first great division 40 upon the dial, where the index then points; hang other 40 lb. to the rope, and mark the division 80, where the index points, and so on, until the index moves round the circle. These great divisions may each be subdivided into 10 small divisions, to indicate 4 lb. each, and can be estimated by the eye to 1 pound. If the spring be properly made, the divisions will be equal; if not, it is a matter of indifference.

The machine may now be fixed to the front of the carriage, and the horse attached to the rope R. The index will at all times point out the friction of the carriage, or the strength exerted by the animal, from nothing up to 400 pounds, with the greatest nicety.

Instruments upon this principle have already been employed for this purpose; but they wanted the fly-wheel; without which they are useless; for without the fly-wheel they are so unsteady in their operation, by reason of sudden jerks and relaxations, that it is impossible to know what the index indicates at a mean.

But, in this case, the fly acts as a steadier and regulator of the force; as it always takes some time to put it in motion, it prevents the influence of sudden changes of force upon the instrument, while, at the same time, it allows it to indicate any permanent change.

This instrument, upon a smaller scale, is per-

fectly adapted for measuring a ship's way at sea, and for ascertaining lee-way.

The friction of wheels is as the diameter of the axle directly, and as the diameter of the wheel inversely; because the friction is overcome by the power of a lever, whose arms are the semidiameters of the wheel and the axle.

This also demonstrates the efficacy of properly-constructed friction-rollers, for diminishing the effect of friction.

Large wheels in carriages surmount all obstacles upon a road easier than small ones.

Demonstration. Let (fig. 44,) there be two wheels, whose centres are A and B, and let C be an obstacle to be surmounted. The weight may be conceived to be collected into the centres A and B, and, in turning round the fulcrum C, the weights will describe arcs, whose centre is C; and, in the first instance, they must ascend the tangents BG and AG of the arcs; but the inclination of AG is greater than that of BG, by angle ACB, and therefore worse to surmount.

RIGIDITY OR STIFFNESS OF ROPES.

(FROM COULOMB'S EXPERIMENTS.)

THE stiffness of ropes increases, and their strength diminishes, the more they are twisted. The twist of a rope therefore ought to be no more than that a single fibre will break rather than be drawn out.

Their stiffness increases from the simple ratio to the duplicate ratio of their diameters, according to the degree of their flexibility.

Their stiffness is as their tensions directly, and as the diameters of the pulleys or cylinders round which they are coiled inversely.

The stiffness of ropes increases but very little with the velocity of the machine.

The stiffness of small ropes is diminished with moisture, but increased in thick ropes.

Professor Leslie has deduced the following rule from the experiments of Coulomb :

Let D express the diameter of the cylinder or pulley in inches, d that of the rope, and P the weight applied to it in pounds avoir. ; then, $d^{\frac{5}{3}}\left(\frac{140+3P}{10D}\right)$, = the stiffness of a new hemp rope, and in old ropes substitute $\frac{7}{3}$ for the exponent of d.

In words. To 3 times the weight add 140, di-

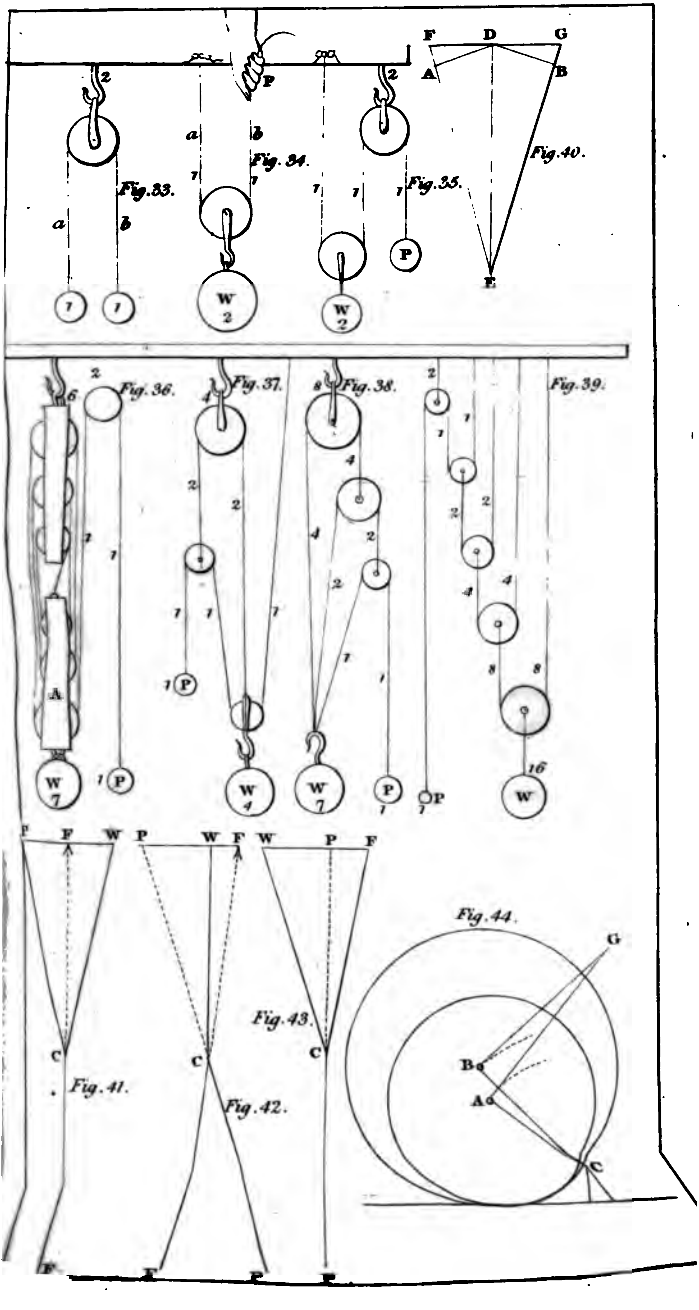


Fig. 33.
Fig. 34.
Fig. 35.
Fig. 36.
Fig. 37.
Fig. 38.
Fig. 39.
Fig. 40.
Fig. 41.
Fig. 42.
Fig. 43.
Fig. 44.
F
D
G
A
B
E
P
W
C

vide the sum by 10 times the diameter of the pulley, and multiply the quotient by the cube root, of the fifth power of the diameter of the rope, for new ropes; but, for old ropes, multiply by the 5th root of the 7th power of the diameter of the rope.

THE CENTRE OF GRAVITY.

PROPOSITION XXXII.

EVERY body in nature has a centre of gravity.

The centre of gravity of a lever of the first order, or of an indefinitely small inflexible rod, is the point where it rests on its fulcrum, (by the definition of the centre of gravity;) for, let the lever be put into any position whatever, the ratio of the forces will not be altered, and their directions remain the same. A surface may be composed of an indefinite number of such rods, whose centres of gravity are in the same straight line, and the centre of gravity of such surface must be somewhere in that straight line; for all the weight may be conceived to be collected into that line, and the line balanced as before; and a solid may be composed of an indefinite number of these thin plates, placed with their centres of gravity in the same straight line; and the centre of gravity of the solid, for the same reason, must be somewhere in

that line: thus a line, a certain surface, and solid, have each a centre of gravity; and, by extending the reasoning, we may show that all bodies have such.

Cor. 1. All the weight of a body may be conceived to be concentrated into its centre of gravity.

Cor. 2. The centre of gravity is the true place of the body; and if the centre of gravity be supported the whole will be supported.

PROPOSITION XXXIII.

If a line drawn perpendicular to the horizon, through the centre of gravity of a body, fall within the base on which it rests, the body will stand; but if the line fall without the base, the body will fall.

Demonstration. (Fig. 47.) Let the perpendicular CF, fall within the base of any body A; the whole weight of the body is supported at C, the centre of gravity, upon the perpendicular CF; but CF cannot move towards E, unless the centre of gravity in the first instance ascend the tangent CE of an arc whose centre is B, which it cannot do without some external force; for the same reason it cannot move towards D, it must therefore stand.

Let the perpendicular fall without the base, (fig.

48,) then the centre of gravity C, must descend the inclined plane CH, having nothing to support it upon the plane.

Cor. 1. The perpendicular drawn through the centre of gravity of the body C, (fig. 49,) may fall within the base; and if a weight D be put upon the top of the body, it may cause the perpendicular to fall without the base, when the whole will fall.

Cor. 2. The larger the base is on which the body rests, the further within it the centre of gravity is, and the lower it is down the firmer the body will stand.

PROPOSITION XXXIV.

If a heavy body AB, (fig. 50, 51,) be suspended by two ropes FA and GB, a plumb-line suspended through P, (the point where the ropes would intersect,) will pass through the centre of gravity C of the body.

Demonstration. Because the forces acting upon the ropes are balanced by gravity, which acts upon the centre of gravity C, the true place of the body, perpendicular to the horizon. But three balancing forces tend to the same centre, that centre must therefore be in the perpendicular PC.

Cor. 1. Draw CH parallel to AF. Then the

weight of the body, the force on GB, and the force on AF, are respectively proportional to the lines PC, PH, and CH, or as the sines of the angles APB, CPA, CPB.

Demonstration. Because the sides of the triangle are respectively parallel to the direction of the forces.

Cor. 2. The strain upon the ropes is greater than the whole weight, when they are in an oblique position.

Cor. 3. When the ropes are parallel, their respective strains are inversely as their distances from the centre of gravity, and are together equal to the whole weight.

Demonstration. Because the sines of the very small angles APB, CPA, CPB, are respectively as AB, CA, CB.

Cor. 4. When the ropes are in the same straight line, their strains are infinitely great. It is therefore impossible to stretch a heavy rope or chain into a straight line unless it hang vertically.

PROPOSITION XXXV.

(Fig. 52.) To find the centre of gravity of any plane, figure ABC.

Suspend it by the string ABD, and mark the direction of a plumb-line DC upon it, suspend the figure again by another loop of the string as E, and the point where the directions of the plumb-line intersects is the centre of gravity.

PROPOSITION XXXVI.

(Fig. 53.) To find the common centre of gravity of any two bodies A and B.

The common centre of gravity is in the straight line joining their centres; and as the sum of the weights of the bodies, A+B : AB : : weight A : BC : : B : AC, where C is the common centre of gravity; which is evident from the principles of the lever.

Cor. To find the common centre of gravity of three or more bodies.

Find the centre of gravity of any two as above, and take the point C, as the sum of these bodies; find the centre of gravity between it and any one of the rest, and so on, for any number of bodies.

The centre of gravity of the following planes and solids has been determined:

In a triangle, the centre of gravity is two-thirds of a straight line, bisecting any one of the sides from the opposite angle.

For a trapezium, draw the two diagonals, and find the centres of gravity of each of the four triangles, whose base is a whole diagonal; then draw

the two lines joining the centres of gravity of each opposite pair; the intersection of these lines is the centre of gravity.

The centre of gravity of a straight line, cylinder, prism, sphere, cube, or regular body, is in the middle.

For the arc of a circle, $\frac{1}{2}$ the arc : sine of $\frac{1}{2}$ the arc : : radius : to the distance of the centre of gravity from the centre of the circle.

For the sector of a circle. As the arc : is to the chord : : $\frac{2}{3}$ radius : to the distance of the centre of gravity from the centre of the circle.

For the parabolic space, the centre of gravity is $\frac{3}{5}$ the axis from the vertex.

For the cone and pyramid, the centre of gravity is $\frac{3}{4}$ the axis from the vertex.

For the paraboloid, the distance is $\frac{2}{3}$ the axis from the vertex.

For the segment of a sphere, let $r =$ radius, $h =$ height of the segment; then the distance of the centre of gravity from the vertex is $\frac{8r - 3h}{12r - 4h}h$.

PROPOSITION XXXVII.

If two bodies move uniformly in straight line, in any direction, their common centre of gravity will either be at rest, or move uniformly in a straight line.

Demonstration. 1. If the bodies move in the

same straight line, since they approach or recede from one another uniformly, their common centre of gravity must approach or recede in the same ratio, and therefore uniformly.

2. Let the bodies move simultaneously from the same point A, (fig. 54,) with the velocities, and in the directions AB, AC; and when in B and C, let their common centre of gravity be G. When the body B is in any point *b*, the body C will be in a point *c*, dividing the lines AB, AC, in the same ratio. Let *g* be the centre of gravity when the bodies are in *b* and *c*, and *n* the intersection of AG and *bc*. Now the body B : body C : : CG : BG : : *cg* : *bg*, by the nature of the centre of gravity; and CG : BG : : *cn* : *bn* by similar triangles; *g* must therefore fall upon *n*, always in the straight line AG, and since it moves along that line, in a constant ratio with the bodies, its motion must be uniform.

3. If the bodies move from different points H and E, (fig. 55, 56, 57,) with the velocities, and in directions HI and EF. Let A be their common centre of gravity when at H and E. If equal bodies were to move simultaneously from the point A, in parallel directions with HI and EF, and with equal velocities to the bodies moving in HI and EF, viz. AB and AC. Let G be their common centre of gravity when in B and C, their com-

mon centre of gravity would move uniformly along the line AG; and when one body is in any point *b*, in the line AB, the other will be in a similar point *c* in the line AC. Let the lines AG, *bc* intersect in *n*, that point has been shown to be the common centre of gravity when the bodies are in *b* and *c*; draw *bi* and *cf* parallel to HE, and join *if*. Now when the bodies moving in the lines AB and AC are in *b* and *c*, the bodies moving in the lines HI and EF are in the points *i* and *f* respectively. Let *g* be their common centre of gravity when in *i* and *f*, then by the nature of the centre of gravity $nb : nc :: gi : gf$, but the triangles *ibn*, *fcn*, are similar; therefore $nb : nc :: ni : nf$, the point *g* the centre of gravity of the bodies, must therefore always fall on *n*, in the straight line AG, and, since it moves along that line in a constant ratio with the bodies, its motion is uniform.

4. The same holds true, when the bodies do not move in the same plane; for the triangles *ibn*, *fcn*, are still in the same plane, and their common vertex *n* always in the line AG.

Cor. If any number of bodies move uniformly in the same straight line, their common centre of gravity will either be at rest, or move uniformly in a straight line.

Demonstration. For any two may be put into

the place of their centre of gravity, and their centre of gravity with a third body will move in a straight line. Put these three into the place of their centre of gravity, and that with a fourth body will still move uniformly in a straight line, and so on.

PROPOSITION XXXVIII.

The state of motion or rest, of the centre of gravity, of any system of bodies cannot be altered by any action of the bodies amongst themselves, or by any force they exert upon one another.

Demonstration. For the centre of gravity is the true place of the body or system, where its whole mass may be conceived to be concentrated; and no body can alter its own state of motion or rest, without the agency of some external cause. The motion or rest of the centre of gravity therefore cannot be disturbed without such external agency.

COLLISION OF BODIES.

PROPOSITION XXXIX.

If two perfectly soft or coalescent bodies A and B, (fig. 58,) move simultaneously in the same direction, and with the velocities represented by AO and BO, A will evidently overtake B in the point

O, where they will unite, and be carried forward in the same straight line. Let G be the common centre of gravity of the bodies, this point will always reach the point O, when the bodies unite there; and the joint mass will move on with the common velocity of the centre of gravity GO, (Prop. XXXVIII.) and B will have exactly gained the quantity of motion that A has lost.

Cor. If the two bodies move in opposite directions, with the velocities AO and BO, (fig. 59,) let G be their common centre of gravity, the bodies will unite in O, and move on with the motion of their common centre of gravity GO, and in that direction.

PROPOSITION XL.

If a perfectly elastic ball A, (fig. 60,) strike another perfectly elastic ball B, while moving in the same direction, their sides at the moment of contact will be partially flattened, and in that instant will move with the common velocity GO of their centre of gravity; and then by their elastic effort will recoil from one another, and the loss of the motion of A, and the gain of the motion of B, are each doubled; or the velocity of A is $= AO - 2AG$, and that of $B = BO + 2GB$, make $GP = GO$, then PA and PB will represent the acquired velocities of A and B respectively.

Cor. 1. If the balls A and B, (fig. 61, 62,) meet in opposite directions at O, with the velocities AO and BO, let G be their centre of gravity, and make GP = GO, in the same manner it may be shown, that the ultimate velocities of A and B will be represented by PA and PB.

Cor. 2. If the ball A, (fig. 63, 64,) strike another B, while at rest, with the velocity and in the direction AO, O must then fall on B, let G be their centre of gravity, and make GP = GO, then the acquired velocities of A and B will still be represented by PA and PB, and in these directions.

Cor. 3. In the figures it may be seen, (fig. 65, 66,) that when the striking ball is the greater, it still advances; when less, it retires; and when equal, it communicates all its motion to the other, and remains at rest.

PROPOSITION XLI.

If a perfectly elastic ball A, (fig. 70, 71,) moving with the velocity and in direction CA, strike another perfectly elastic ball B obliquely, in the point D, the plane of their collision LM will evidently be perpendicular to a straight line passing through the centres of both. Draw CN and AF parallel, and HDB perpendicular to LM; make HN and AF each equal to HC, and join NFI; divide the line HB, so that HG : GB :: ball B : ball A, G would

then be the centre of gravity, if the ball A were at H. Make GP = GD, and AE = HP, draw EI parallel to AF, and join AI. Suppose the force CA, decomposed into the two forces CH and HA, the force CH has no effect upon the collision, while the force HA is directly against the ball B, and the force CH is still continued towards N; therefore PH represents the force of A's recoil, from B, AI its velocity and direction, and PA is B's velocity and direction.

Cor. 1. When the balls are equal, P falls in H, E in A, and I in F; and they have the property, that, with whatever force they strike, or in whatever oblique direction, they recede from one another in the two sides of a rectangle.

Cor. 2. When, instead of the ball B, let A strike an object of indefinite weight, as a wall, then G falls on A, E in H, and I in D; and the ball A moves along the diagonal AI, with an angle of reflection equal to the angle of incidence.

Upon the above two propositions, and their corollaries, depends the theory of the games of billiards, curling, golf, &c.; but as there are no bodies perfectly elastic, that come under this consideration, nor free from friction, the angles of reflection, and also the velocities, will be less than those given in the theory.

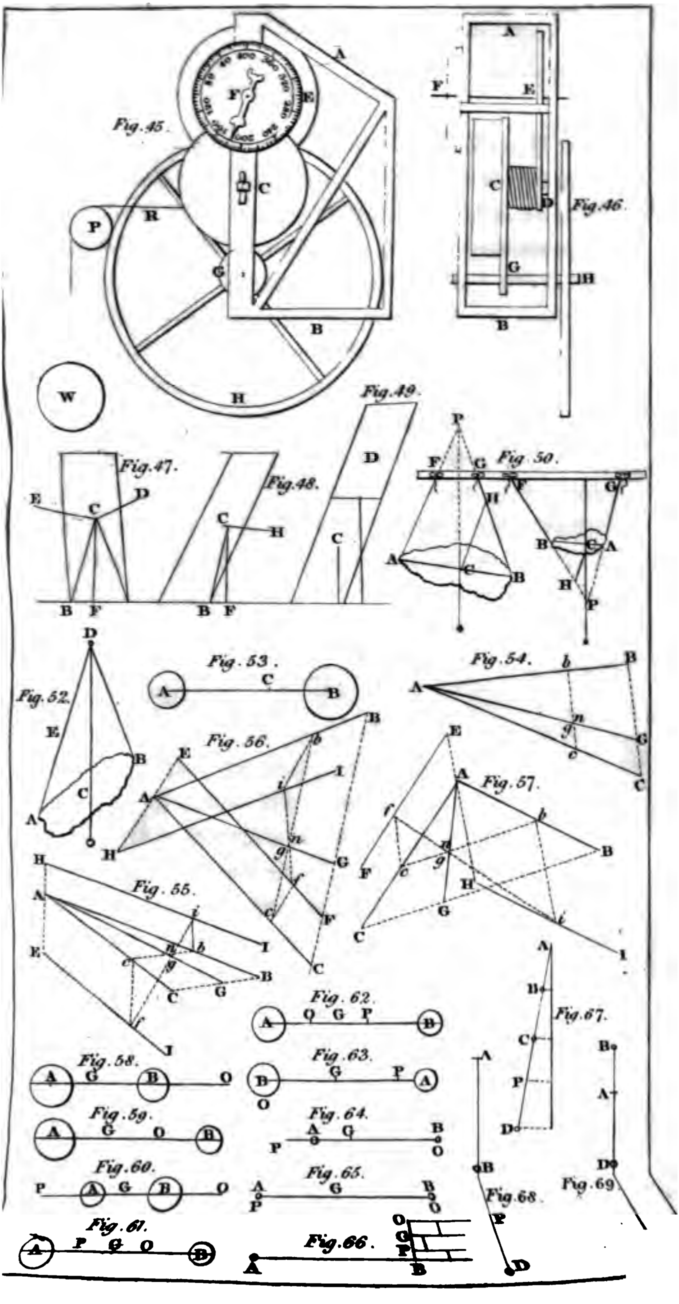


Fig. 45.
Fig. 46.
Fig. 47.
Fig. 48.
Fig. 49.
Fig. 50.
Fig. 52.
Fig. 53.
Fig. 54.
Fig. 55.
Fig. 56.
Fig. 57.
Fig. 58.
Fig. 59.
Fig. 60.
Fig. 61.
Fig. 62.
Fig. 63.
Fig. 64.
Fig. 65.
Fig. 66.
Fig. 67.
Fig. 68.
Fig. 69.

products, by the sum of the products of the weights into their distances simply.

Demonstration. Let B, C, and D, (fig. 67,) be a system of bodies suspended from A, to vibrate by an inflexible rod, and let P be the centre of oscillation, the bodies B, C, have a tendency to accelerate the motion (Prop. XXIII. Cor. 1.) of the point P, while D has a tendency to retard that motion. Let AP express the accelerative force at P; BP and CP will express the excess of acceleration at B and C, and DP the retarding force at D. Now from the principles of the lever, the effects of these forces must be as their distances from the point of suspension, and the accelerating force at P, must be equal to the retarding force. Therefore $B\times BP\times AB+C\times CP\times AC=D\times DP\times AD$, but $PB=AP-AB$, $CP=AP-AC$, and $DP=AD-AP$, and consequently $B\times(AP-AB)\times AB+C\times(AP-AC)\times AC=D\times(AD-AP)\times AD$, or $AP=\frac{B\times AB^2+C\times AC^2+D\times AD^2}{B\times AB+C\times AC+D\times AD}$.

Cor. 1. If any of the bodies be placed above the point of suspension, the product of their weights and distances must be taken negative.

Cor. 2. If the bodies are not in the same straight lines, or even not in the same plane, draw

a line from the point of suspension through their common centre of gravity; draw perpendiculars from each of the bodies upon that line; then the sum of the rectangles made by each of the bodies into the distance of that perpendicular from the point of suspension must be taken for the divisor.

The above equation furnishes the following rule, as well as many others, that may be useful in compound pendulums, viz.

Having the distances of two weights B and D, (fig. 68, 69,) from the centre of suspension of a compound pendulum; to find the ratio of the weights, so that the pendulum may beat seconds, or any given time equal to a simple pendulum, whose length is AP.

Rule. Case 1st. When the weights are both below the centre of suspension. Find the length AP of a simple pendulum that will vibrate the given time, (Prop. XXIII. Cor. 3.) Then take the difference of the square of the distance of one of the weights from the centre of suspension, and the product of that distance, multiplied by the length of the simple pendulum, for a dividend. Take the difference of the square of the distance of the other weight, and the product of that distance multiplied by the length of the simple pendulum for a divisor. Find the quotient, and as 1 : quotient :: first weight : second weight.

Case 2d. When one of the weights is above the

point of suspension. Proceed as in case 1 for a dividend. But take the sum of the square and product for a divisor, instead of the difference. Find the quotient, and as 1 : quotient : : first weight : to the second weight.

EXAMPLE I.

In a compound pendulum, (fig. 68,) the distance AD is 4 feet, and AB 2 feet; required the ratio of the weights D and B, that the pendulum may beat seconds.

The length of a seconds pendulum being 39·1386 inches, or 3·2615 feet; therefore $\frac{4^2 - 3{\cdot}2615 \times 4}{3{\cdot}2615 \times 2 - 2^2} = \frac{16 - 13{\cdot}046}{6{\cdot}523 - 4} = \frac{2{\cdot}954}{2{\cdot}523} = 1{\cdot}1708$; and as 1 : 1.1708 : weight D : to weight B; where either D or B may be any weight at pleasure.

EXAMPLE II.

In a compound pendulum, (fig. 69,) the distance AD is 1 foot, and the distance AB ·8 feet, above the point of suspension; required the ratio of the weights D and B, so that the pendulum may beat seconds.

$\frac{3.2615 \times 1 - 1^2}{{\cdot}8^2 + 3{\cdot}2615 \times {\cdot}8} = \frac{2{\cdot}2615}{3{\cdot}2492} = {\cdot}696$; and 1 : ·696 : : weight D : weight B.

The intelligent reader will easily perceive the limits within which the problem is possible in a given case.

Investigation of the Rule.

Since $AP = \frac{B\times AB^2+C\times AC^2+D\times AD^2, \&c.}{B\times AB+C\times AC+D\times AD, \&c.}$, by expunging the quantities connected with C, and making $D = 1$, we have $B = \frac{AD^2 - AP\times AD}{AP\times AB - AB^2}$ for case 1st, and $B = \frac{AP\times AD - AD^2}{AB^2 + AP\times AB}$ for case 2d.

DEFINITION.

The centre of percussion in a body, or system of bodies, revolving round an axis, is a point in it, which striking an immoveable object, the body or system shall incline to neither side, but remain at rest in equilibrio.

PROPOSITION XLIII.

The centre of percussion is the same as the centre of oscillation.

Demonstration. Let A (fig. 67,) be the axis of motion, and P the centre of percussion, which in striking an immoveable object the same reasoning holds when applied to the forces of B, C, and D, to turn the system round the point P, as was applied to those bodies, accelerating and retarding the motion at that point, in oscillating.

When a man tries to break a staff, by striking the point against the ground, he receives a very disagreeable wrench in the hand; because, in that case, his hand receives a considerable portion of

the blow; but if he strikes at the centre of percussion of the staff, (his wrist being supposed the centre of motion,) his hand will not receive the smallest concussion. All tools that work by percussion ought therefore to have a portion of their weight beyond the striking point, in order to save the hand.

The distance of the centre of oscillation and percussion, from the centre of suspension, has been determined as follows, when the axis of motion is in the vertex of the figure, and in the same plane:

In a triangle $\frac{3}{4}$ the axis.

In a straight line, small parallelogram, or cylinder, $\frac{2}{3}$ the axis.

In a circle, $\frac{5}{8}$ the diameter.

In a parabola, $\frac{5}{7}$ the axis.

In a pyramid, or cone, $\frac{4}{5}$ the axis.

In a sphere, let r = radius, d = distance of the axis of motion from the centre, $d+\frac{2r^2}{5d}$.

PRACTICAL REMARKS ON THE CENTRE OF GYRATION.

The centre of gyration of a system of bodies revolving about a fixed axis is a certain point, in which, if the whole weight of the system were

placed, the same angular velocity would be generated in that weight, as in the system, with an equal impulse applied at equal distances from the centre of motion.

The centre of gyration of a body, or system of bodies, is a mean proportional between the distance of the centres of gravity and of oscillation from the centre of motion. As the demonstrations of the centre of gyration are tedious and intricate, they are here omitted.

The distance of the centre of gyration, of the following bodies, from the axis of motion, has been ascertained to be,

1. In a straight line, or small cylinder, revolving about one end. The length multiplied by ·5775.

2. A cylinder, or plane of a circle, revolving about the axis. The radius multiplied by ·7071.

3. The circumference of a circle about the diameter. The radius multiplied by ·7071.

4. The plane of a circle about the diameter, half the radius.

5. The surface of a sphere about the diameter. The radius multiplied by ·8165.

6. A globe revolving about the diameter. The radius multiplied by ·6324.

7. The circumference of a circle, upon a perpendicular axis, passing through its centre. The whole radius.

The further the centre of gyration is from the

centre of rotation, it requires the greater force or time to generate a given velocity in the revolving body; as also to destroy that motion when once generated. For this reason a fly-wheel is the more efficient, the more of its weight lying in its rim, and the less in its centre, as having then the greatest force with the least weight upon its axis, and consequently less friction and expense of material.

PROBLEM.

Given the weight of a revolving body, the distance of its centre of gyration from the axis of rotation; and a constant force acting against a given point; to determine the time in which the body will acquire a given velocity.

All constant forces may be compared to gravity. Therefore, as the weight of the body to be moved : is to the effective weight or force moving it : : so is the length of an inclined plane : to its perpendicular height. This plane is such, that the given weight or force would support the body upon it. The problem is therefore reduced to this. To find the time in which a body would acquire the given velocity, descending down the given plane.

EXAMPLE I.

A fly-wheel, 4 feet 3 inches radius, weighing 3000 pounds, is driven by 2 men exerting together 60 pounds, upon a crank 1 foot 4 inches from the centre of rotation; the wheel's centre of gyration is

4 feet from its centre. In what time will its centre of gyration acquire a velocity of 20 feet per second?

By the principles of the lever 4 feet : 1 foot 4 inches :: 60 : 20 lb., the force exerted at the centre of gyration.

It is now required to know the time in which a body would acquire a velocity of 20 feet per second on an inclined plane, whose length and height are in proportion to 3000 and 20.

By Prop. XIV. $\frac{20}{32{\cdot}19} = {\cdot}6213$ seconds, the time falling perpendicularly.

By Prop. XXII. Cor. 2, 20 : 3000 : : ·6213 : 93·19 = 1 minute 33·19 seconds, the time required.

EXAMPLE II.

In what time would the same force, upon the same wheel, generate a quantity of motion in it equal to that of a 24-pound cannon-ball, projected with a velocity of 3000 feet per second?

As the weight of the wheel 3000 : is to the weight of the ball 24 : : so is the velocity of the ball 3000 : to the velocity of the wheel 24; then, as before, $\frac{24}{32{\cdot}19} = {\cdot}7456 =$ the time falling perpendicularly; and 20 : 3000 : : ·7456 : 111·84 = 1 minute 51.84 seconds.

If it were possible to apply the force of the wheel to the ball, in a suitable manner, by means

of a lever or otherwise, it would in this case project the 24 lb. ball with the velocity of 3000 feet per second, and would then stop all its own motion.

To find the velocity of the circumference of the wheel.

As the distance of the centre of gyration from the centre of rotation : is to the radius of the wheel : : so is the velocity of the centre of gyration : to the velocity of the circumference.

Note.

Let G = the distance of the centre of gyration from the axis of rotation.

W = the weight of the wheel.

F = the force acting upon it.

D = the distance of that force from the axis of rotation.

t = the time the force acts.

v = the velocity generated in the time t.

$a = 32.19$.

Then we have the following equations, comprehending all the rules concerning this kind of rotatory motion :

$F = \frac{GWv}{Dta}$, $D = \frac{GWv}{Fta}$, $G = \frac{FDta}{Wv}$, $W = \frac{FDta}{Gv}$, $t = \frac{GWv}{FDa}$, $v = \frac{FDta}{GW}$.

It is evident, from the foregoing calculations, that the fly-wheel is not only a regulator of motion, but also a concentrator of force; and hence arises its great utility, in its application to cutting strong

cold iron plates, in punching holes through them, and in many other purposes to which it is applied.

What has been said of rotatory motion, is also applicable to rectilinear motion; the velocity of the body itself being substituted for the velocity of the centre of gyration.

The above calculations elucidate, that all motions must commence with an accelerated and terminate with a retarded motion: That no body can acquire a given velocity instantaneously, nor when acquired have that motion stopped instantaneously, although the time is so short, in many cases, as to escape our observation.

That therefore all machines contrived for producing an easy reciprocating motion, should commence and terminate the motion by slow degrees, having their greatest velocity in the middle of the space, where the motion begins and ends, and not with a uniform motion, as some authors affirm, or it will shake or break the machine to pieces. Nature points out the propriety of this, in the vibration of a pendulum, the ebbing and flowing of the tides, &c. An easy commencement and termination of the motion of the working-beam of a steam engine is naturally preserved by the elasticity of the steam in the cylinder, and an instantaneous commencement of the motion of a ball from a gun is prevented naturally by the elasticity of the ignited vapour in the barrel. When the machine is contrived to act by percussion, this rule does not

hold of course; for then its purpose is to break or be broken.

From the above considerations, we easily perceive the reason of the great power of percussion, compared to mere pressure. Percussion is a force endeavouring to destroy a generated motion instantaneously, which is impossible, and therefore its advantage above mere pressure is evident; for in this there is no motion to destroy, but only an endeavour to propagate motion.

The simple pressure of ten men will not drive a nail that one man may do with a small hammer easily. The reason is, that in the first case there is no motion to destroy, and in the second case the nail is driven, because the motion of the hammer cannot be instantly destroyed.

CENTRAL FORCES.

When a body is compelled to revolve in a circle by any force drawing it towards the centre, that force is called the centripetal force. The force opposing this, or that whereby the body endeavours to move forward in a straight line in the tangent of the circle, is called the centrifugal force, and the two together are called central forces. They are

equal, and therefore what is said of the one applies equally to the other.

As the demonstrations of central forces are tedious and intricate, they are omitted here; and we shall introduce a comprehensive theorem by the Marquis de l'Hôpital, namely,—

Find from what height the body must have fallen to acquire the velocity in the circle; then, as the radius of the circle : is to twice that height :: so is the weight of the body in motion : to its centrifugal force. From which theorem the following equations have been derived:

Let v = the velocity of the body;
r = the radius of the circle;
w = the weight of the body;
f = the centrifugal force; and
32 = the velocity acquired by a body falling in 1 second. Then, $f = \frac{v^2 w}{32r}$, $r = \frac{v^2 w}{32f}$, $w = \frac{32fr}{v^2}$, $v = \frac{32fr}{w}$.

In words. 1. To find the centrifugal force. Multiply the square of the velocity by the weight of the body, and divide by 32 times the radius.

2. To find the radius. Multiply the square of the velocity by the weight, and divide by 32 times the centrifugal force.

3. To find the weight. Multiply 32 times the

centrifugal force by the radius, and divide by the square of the velocity.

4. To find the velocity. Multiply 32 times the radius by the force; divide by the weight, and extract the square root of the quotient.

EXAMPLES.

1. Required the centrifugal force of the rim of a fly-wheel, or the force by which it endeavours to burst asunder, the radius being 6 feet, and the weight of the rim 1 ton, making 65 revolutions in a minute.

$\frac{12 \times 3{\cdot}1416 \times 65}{60}, = 40{\cdot}84 =$ the velocity in feet per second. Then, by rule 1. $\frac{40{\cdot}84^2 \times 1}{32 \times 6} = \frac{1667{\cdot}9056}{192} =$ 8·687 tons, Answer.

2. The above fly-wheel is in two halves, joined together by bolts, which in their various positions are capable of supporting 4 tons, its whole weight is $1\frac{1}{2}$ tons, the circle of gyration is $5\frac{1}{2}$ feet from the axis of motion; required its velocity, such that the two halves may burst asunder.

It is not difficult to perceive, that since the centrifugal force is exerted equally all around, the force tending to separate the two halves is one-half the whole force; therefore, by rule 4,

$$\sqrt{\frac{32 \times 4 \times 5{\cdot}5 \times 2}{1{\cdot}5}} = \sqrt{\frac{1408}{1{\cdot}5}} = 30.638 = \text{velocity}.$$

$11 \times 3{\cdot}1416 = 34{\cdot}5576 =$ the circumference of circle.

$34{\cdot}5576 : 30.638 :: 60 : 53{\cdot}195 =$ revolutions per minute.

THE QUANTITY AND DIRECTION OF PRESSURE,

AND

THE THEORY OF ARCHES.

PROPOSITION XLIV.

If a beam be supported at A and C, (fig. 73, 74, 75, 76,) lying upon or leaning against a wall, or other immoveable body BC, with one end; and G the centre of gravity of the beam, and the weight that may be upon it. Draw the line EGF perpendicular to the horizon, and CE and AF perpendicular to BC; then

The whole weight is in proportion to EF.

The pressure at the top C to AF.

The thrust or pressure at the base A to EA, and in these directions.

Demonstration. Because the direction of the whole weight of the beam in the centre of gravity

is EF, and the pressure at C, in direction EC, (Prop. XX.) The direction of the pressure at A must therefore be EA, (Prop. XIII.) But AF is parallel to EC; the sides EF, EA, and AF, are therefore in the directions of the balancing pressures, and must be in proportion to them, (Prop. IX.)

Cor. 1. Resolve the force EA into the two forces EH and HA, (Prop. X.) then the perpendicular pressure at A is EH, and the thrust outward HA. Resolve the force AF into the two forces AH and HF; then the perpendicular pressure at A is HF, and the thrust outward AH.

Cor. 2. The horizontal pressure at top and bottom is always equal, and in opposite directions.

Cor. 3. The horizontal pressure is greater the higher the centre of gravity is, and whence the higher a man ascends a ladder so placed, the greater the danger of it being thrust out at bottom.

Cor. 4. When BC inclines, the perpendicular pressure of the beam at top is in direction downward; when BC is vertical, the horizontal pressure at top is nothing; and when it reclines, the perpendicular pressure at top is in direction upward.

Cor. 5. The weight, pressures at A and C, are respectively as the sines of AEC, GEC, and AEG.

PROPOSITION XLV.

If a heavy beam be sustained at C, (fig. 77, 78,) and moveable about a point there, while the other end B lies loosely upon a wall AD. Draw EGF through the centre of gravity G of the beam and weight upon it, perpendicular to the horizon, and EC, AF, perpendicular to the surface at A; then

The whole weight is in proportion to EF.

The force acting at C to CF.

The pressure at A to EC, and in these directions, in the case of fig. 77, but reversed in fig. 78.

Demonstration. Because (fig. 77,) the direction of the weight, in the centre of gravity is EF, and the pressure at A is in direction AF, (Prop. XX.) the direction of the force at C is therefore CF, (Prop. XIII.); but EC is parallel to AF; the three sides of the triangle EF, EC, and CF, therefore represent the balancing forces. The case of fig. 78 is similarly demonstrated.

Cor. 1. If the end A lies upon the horizontal plane AB, the lines AF and EF are parallel, and consequently the line CF; therefore the weight, pressure at A, and force at C, are respectively pro-

portional to AC, GC, and AG, and the horizontal pressures are nothing.

Cor. 2. HC represents the lateral force at A and C, EH and HF the perpendicular forces at A and C respectively.

Cor. 3. The weight, pressure, or force at A and C, are as the sines of the angles AFC, GFC, and GFA, respectively.

PROPOSITION XLVI.

If a heavy beam DE, (fig. 79,) whose centre of gravity is G, be supported upon two posts AD and BE, and let the whole be moveable about the points A, D, E, B; let AD and BE produced meet in any point C of a perpendicular to the horizon, drawn through G, the centre of gravity; draw FH parallel to AC; then

The weight of the beam is in proportion to CF.
The pressure at E to CH.
The pressure at D to HF, and in these directions; also the beam cannot be supported in any other position.

Demonstration. For the pressure at D and E is the same as though the beam were suspended by a rope from C; therefore, (Prop. XXXIV.

Cor. 1,) the weight, pressures at D and E, are respectively as CF, HF, and CH; and three forces that balance one another must tend to the same point, which must be in the line CG.

Cor. The weight, pressure upon D and E, are respectively as the sines of the angles DCE, ECG, and DCG.

PROPOSITION XLVII.

If several beams AB, BC, CD, &c. be joined together at B, C, D, &c. (fig. 80,) in a vertical plane, and moveable about the points A, B, C, &c. the points A, F at the extremities being fixed; through the angles at B, C, D, &c. draw *ri*, *sm*, *tp*, &c. perpendicular to the horizon. Then, when the beams support one another in equilibrio, the weight upon the several angles must be in proportion to the sine of the angle made by the beams, divided by the product of the cosines of their inclination above the horizon. That is, $\frac{\text{sin. BCD}}{\text{sin. BC}m \times \text{sin. DC}m}$ is as the weight upon the angle C.

Demonstration. Because sin. ABC : sin. ABr :: weight B : force in direction BC $= \frac{\text{B} \times \text{sin. AB}r}{\text{sin. ABC}}$ (Prop. XII.), and sin. BCD : sin. DCs :: weight C : force in direction CB $= \frac{\text{C} \times \text{sin. DC}s}{\text{sin. BCD}}$. But these two forces must be equal to preserve the equili-

brium; that is, $\frac{B\times\sin.ABr}{\sin.ABC}=\frac{C\times\sin.DCs}{\sin.BCD}$, consequently $B:C::\frac{\sin.ABC}{\sin.ABr}:\frac{\sin.BCD}{\sin.DCs}$; and for the same reason, $C:D::\frac{\sin.BCD}{\sin.BCs}:\frac{\sin.CDE}{\sin.EDt}$; therefore weight B : weight D : : $\frac{\sin.ABC}{\sin.ABr\times\sin.BCs}:\frac{\sin.CDE}{\sin.DCs\times\sin.EDt}::$ $\frac{\sin.ABC}{\sin.ABi\times\sin.CBi}:\frac{\sin.CDE}{\sin.CDp\times\sin.EDp}$.

Cor. 1. Draw C*p* parallel to DE, and D*m* to CB, then, to preserve the equilibrium, the weight on C : weight on D : : C*m* : D*p*; so that when the weights are given, and the position of two beams CD and DE, all the rest may be found in succession, as will be shortly shown.

Demonstration. Let CD represent the force in direction CD or DC, then CP is the force of C, in direction DE, and D*m* is the force of D, in direction CB; but D*p* is the equivalent force of DC and C*p*, and C*m* is the equivalent force of CD and D*m*, (Prop. IX.)

Cor. 2. If the whole figure ABCDEF were turned upside down, and the extremities also inverted, the weights to remain the same, and the points A and F fixed as before, the whole figure would remain the same as before, whether the lines be flexible or inflexible.

Demonstration. Because all the forces would remain as before, but completely inverted. The forces must therefore preserve the equilibrium the same as before.

Cor. 3. When the weights are all equal, (fig. 81,) and the number of angles indefinite, as in the case of a heavy rope, or chain, bent into a curve by its gravity, the curve is called a catenaria. Let AEB be such a curve, and AC and BC tangents to it, complete the parallelogram ACBD. Then DC : AC : : weight of the rope or chain : to the strain at A ; and DC : BC : : weight : strain at B, and DC is perpendicular to the horizon.

Demonstration. Let the lines AC, BC, be supposed without weight, and let a weight equal to the rope be hung at C ; its force will be in direction DC, perpendicular to the horizon ; let DC represent that force, it is supported by two equivalent forces AC, BC, in direction of the tangents ; but the weight at C is the weight of the rope, which exerts its force at A and B, in direction of the tangents ; DC, AC, and BC represent therefore the weight and strains at A and B respectively.

Cor. 4. Professor Leslie's approximate rule for calculating the strain upon the ends of a catenarian arch, also the length of the chain, when the

ends are upon the same level, and having a small depression.

Let B denote the breadth or span of the arch.
D the depth or depression of the arch.
L the length of the chain.
P the strain at the lowest point.
S the strain at the ends.

Then $L = B + \frac{8D^2}{3B}$, $P = \frac{B^2}{8D} + \frac{D}{6}$, and $S = \frac{B^2}{8D} + \frac{7D}{6}$.

Or 1. To the span add 8 times the square of the depression, divided by 3 times the span; the sum is the length of the chain.

2. To the square of the span, divided by 8 times the depression, add $\frac{1}{6}$ of the depression; multiply the sum by the weight of 1 unit of the chain, for the strain at the lowest point.

3. To the square of the span, divided by 8 times the depression, add $\frac{7}{6}$ of the depression; multiply the sum by the weight of 1 unit of the chain, for the strain at the ends.

EXAMPLE.

What are the strains of a catenarian arch, whose span is 300 feet, depression 24 feet, and each foot of the chain weighing 4 pounds?

$300+\frac{24^2\times 8}{300\times 3}=300+\frac{4608}{900}$, $305\cdot 12=$ length of the chain.

$305\cdot 12\times 4=1220\cdot 48=$ weight of the chain.

$\frac{300^2}{24\times 8}+\frac{24}{6}=\frac{90000}{192}+4=471.7$; $471\cdot 7\times 4=1886\cdot 8$ lb. $=$ the strain at the lowest point.

$\frac{300^2}{24\times 8}+\frac{24\times 7}{6}=\frac{90000}{192}+28=495\cdot 7$; $495\cdot 7\times 4$ lb. $=$ the strain at either end; consequently the whole strain on the abutments will be 1982·8 lbs., multiplied by the number of such chains employed.

The same rule holds true, whether the arch be suspended or incumbent; when the latter, take the weight of the materials of the arch instead of the weight of the chains.

PROPOSITION XLVIII.

If the weights are disposed in the arc of a circle (fig. 82,) at equal distances, they are to one another as the square of the secants of the angles of deflection above the horizon at the points where they are placed.

Demonstration. Divide the arch AF into an indefinite number of equal portions, AB, BC, CD, &c. then the angles ABC, BCD, &c. are all equal, and the arc at any point B, makes the opposite angles equal, with a perpendicular rBi; the cosines of the alternate angles at any point B are therefore equal; whence, by the last proposition, the weights

at the several points A, B, C, &c. are inversely as the squares of the cosines of the angles of deflection, the angles ABC, BCD, &c. being constant; but the cosines are as the secants inversely; therefore the weights are as the square of the secants of the angles of deflection directly.

Cor. The weights on the several points of a circular arch must be as the differences of the tangents of the corresponding angles of deflection.

Demonstration. It has been shown, (fig. 82,) that the weight of the portions FE, ED, DC, &c. must be as OF^2, OM^2, ON^2, &c.; OM, ON, OP, &c. being the secants of the several arcs. From C and P draw CR and PS perpendicular to OB, then OC : OP : : CR : PS, and OC (= OF) : OP : : PS : PQ, by similar triangles, whence, by composition, OC^2 or $OF^2 : OP^2$: : CR or FM : PQ, &c.

The above two propositions include the whole theory of arches, whether suspended or incumbent.

In stone arches, small deviations from the theory are compensated by the cohesion of the materials, and the support of large abutments, which in the theory are considered as mere points. In arches of large span, however, the less deviation from the theory the better; for in that case the compensations bear a smaller ratio to the whole.

A Mechanical Method of finding the Weight upon the several Parts of an Arch, or the Thickness of the Material.

Draw an arch upon a large board, of the same form you wish, as ABC, from a large scale, and place it vertically, but inverted. Take a cord calculated of the same length as your drawn arch, divide it into any number of equal parts, to each division fasten hooks of wire, and hang its ends on A and C; hang bundles of wire to each hook, until, by adding or clipping, the points of the cord coincide with the drawn arch; weigh each weight separately; to which add the weight of its share of the cord, and these will give the ratio of the weights that ought to be laid upon the corresponding parts of the arch; so that when you know the thickness or weight upon the crown, you know all the rest.

It is not necessary that the voussoirs, or arch-stones, should be of the weights here given, but that they may be made up to that weight with rubble.

It is the opinion of some authors of eminence, that an arch cannot be supported, unless a catenaria be contained somewhere within it; but it is obvious that this can be true only in those species of arches that have the weight equally distributed over every part of them. But in modern stone

bridges, that have the road-way nearly level, a catenaria is almost the worst possible form, since most of the weight is near the piers and abutments; but a segment of a circle, of a height suitable to the height of the banks of the river, may be shown in most cases to be the best form.

Suspension-bridges will in all cases resolve themselves into the best form of a curve, if they are not otherwise restrained from doing so.

Timber bridges, constructed of several beams, supported by one another, and the whole by the abutments, ought to have a form that is easily deducible from Proposition XLVII., or mechanically, as above, where half the weight of each two adjoining timbers may be taken for the weight upon the angles.

It may be perceived, from the above theory, that the strain upon the abutments is in all cases greater than the whole weight of the arch; and the flatter the arch is, the greater is the strain, whether it be suspended or incumbent; and whence the necessity of a firm foundation for the abutments and piers, and having them so constructed, that they will resist, immoveably, the immense outward pressure,—the mode of performing which belongs to Architecture.

STRENGTH AND STRAIN OF MATERIALS.

Bodies are exposed to four different kinds of strain, viz.

1. They may be torn asunder, as in the case of ropes, stretchers, tie-beams, &c. The strength of a body to resist this kind of strain is called its absolute strength.

2. They may be broken across, as in joists, rafters, levers, &c. The strength of a body to resist this kind of strain is called its lateral strength.

3. They may be crushed, as in the case of pillars, posts, &c.

4. They may be twisted or wrenched, as in the case of axles, screws, &c.

ABSOLUTE STRENGTH.

PROPOSITION XLXI.

The absolute strength of ropes of uniform texture is in proportion to the area of their transverse sections: that is, as the squares of their diameters, let their lengths be what they may.

For it is evident, when a rope is stretched, every section of it must bear an equal strain, because action and reaction are equal; there is no difference, therefore, between the strength of a long rope, and a short one of the same thickness and quality, excepting that a long rope adds its own weight to the strain upon it. And because the cohesion of each fibre or particle is equal, the whole cohesion must be equal to their number; that is, to the area of the section.

Cor. 1. All cylindrical or prismatic rods are equally strong in every part; but bodies that have unequal sections will break where the section is least.

Cor. 2. The above is true, either with ropes, chains, or stretchers of any kind, whether of wood, metal, stone, &c.

TABLE

Of the Absolute Cohesion of Timber, or the Weights that wil tear asunder lengthways a Prism of one Inch square.—*From Professor Leslie's Natural Philosophy.*

	lb.		lb.
Teak	12915	Memel fir	9540
Oak	11880	Christiana deal	12346
Sycamore	9630	Larch	12240
Beech	12225	White marble	1811
Ash	14130	Portland stone	857
Elm	9720	Craigleith stone	772

From the Experiments of Mr Emerson; being the Weight that a square Inch will bear with Safety.

	lb.		lb.
Iron	76400	Red fir, holly, alder, plane, crab	5000
Brass	35600	Cherry, hazel	4760
Oak, box, yew, plum	7850	Alder, ash, birch, willow	4290
Elm, ash, beech	6070	Freestone	914
Walnut	5360		

TABLE

Of the absolute Cohesion of the following Substances, from the Experiments of M. Muschenbroek; being the Weight that will pull a Prism asunder one Inch square.

Cast gold from	20000 to 24000	Bismuth	2900
— silver	40000 to 42000	Good brass	51000
— copper from Anglesea	34000	Ivory	16270
— — from Sweden	37000	Horn	8750
Cast-iron from	42000 to 59000	Whalebone	7500
Bar-iron, ordinary	68000	5 parts gold with 1 of copper	50000
— best Swedish	84000	5 parts silver with 1 of copper	48500
Bar-steel, soft	120000	6 parts Swedish copper, 1 of tin	64000
— razor temper	150000	3 parts block-tin, 1 of lead	10200
Cast-tin, English block	5200	4 parts tin, 1 of lead, and 1 of zinc	13000
— grain	6500	8 parts lead and 1 of zinc	4500
Cast-lead	860		
Antimony	1000		
Zinc	2600		

The same.—*From the Experiments of Mr Rennie.*

	Weight that would tear it asunder.	Length that would break with its own weight.
Cast-steel,	134256 lb.	39455 feet.
Swedish iron,	72064	19740
English do.	55872	16938
Cast-iron,	19096	6110
Cast-copper,	19072	5003
Yellow brass,	17958	5180
Cast-tin,	4736	1496
Cast-lead,	1824	384
Good hemp-rope,	6400	18790
Do. 1 inch diameter,	5026	18790

From M. Muschenbroek's experiments, the reader will see the great augmentation of strength by the several mixtures for pipes, cannon, or vessels requiring strength.

The above numbers are such as will break the bodies in a very short time; the prudent artist therefore will do well to trust no more than about one-third of these weights; also great allowance must be made for knotty timber, and such as is sawn in any part across or oblique to the fibres.

From the tables we can find the weight that any rope, rod of metal, chain, &c. will bear; also what area of section is required to sustain a given weight.

EXAMPLE I.

What weight will a rope 2 inches in diameter sustain?

$5026 \times 2^2 = 20104$ pounds, or 8 tons 19 cwt. 2 quarters.

EXAMPLE II.

What weight will an iron wire $\frac{1}{10}$ inch bear?
$84000 \times \frac{1}{10}^2 \times .7854 = 659\cdot7$ pounds.

EXAMPLE III.

Required the diameter of a rope that will sustain 8 tons, 19 cwt., 2 quarters, or 20104 pounds.

$\sqrt{\frac{20104}{5026}} = 2$ inches diameter required; but for safety we ought not to trust more than one-third of any of these weights.

For an example, we shall here investigate the practicability of Mr Anderson's plan for a suspension-bridge across the Forth at Queensferry, or the strains at the ends of the catenaria.

From the shore to Garvy Island is 2000 feet for the first span, and the depression of the curve is 80 feet.

$\frac{2000^2}{80 \times 8} + \frac{80 \times 7}{6} = 6259\cdot3 =$ the strain at the ends, (Prop. XLVII. Cor. 4,) or the weight of a chain of 6259 feet. Now, from the extra weight occasioned by the ends of the links of the chain, (which contribute nothing to the strength of the section,) we must call the strain at least 8000 feet; and the weight that the chain must necessarily carry cannot surely be less than its own weight, or in all

16000 feet; but 19740 feet of Swedish iron breaks with its own weight, it cannot therefore be trusted with the weight of 16000 feet. In the plan, the chains are stretched into straight lines, which is impossible, (Prop. XXXIV. Cor. 4.)

LATERAL STRENGTH AND STRAIN.

PROPOSITION L.

The strength of any beam of timber, or stone, or bar of metal, to resist a lateral strain is in proportion to its breadth, multiplied by the square of its depth.

Demonstration. Let ABCD be a rectangular beam, (fig. 83,) fixed into a wall at AD, and W a weight hung at the end BC. Let us suppose the beam to be perfectly strong in every part, except in the section EF, and that it will break in that section by turning round the point F, as the fulcrum of a lever whose one arm is CF. Let the line FE be divided in the points 1, 2, 3, &c. into as many equal parts as there are fibres or particles that cohere together in the section FE, call each of these parts 1, and the number of parts n, then the power of the weight acting at C, by the arm of

the lever CF, to overcome the cohesion is, $\frac{0}{FC}+\frac{1}{FC}+\frac{2}{FC}+\frac{3}{FC}+\ldots\frac{n}{FC}$; that is, (by arithmetical progression,) $\frac{n^2}{2FC}=\frac{FE^2}{2FC}$, or as the square of the depth; and it is evident, that in whatever proportion the breadth be augmented or diminished, the strength of the section will also be augmented or diminished, or the number of the levers CFE, and therefore is as the breadth.

We have here supposed, that every fibre in the section is pulled asunder; but this is really not the case; for part of it towards the point F will be crushed inward, and consequently the fulcrum F will be removed to some distance towards E; but, although this circumstance weakens the body, yet the strength will still be the square of the depth, divided by some constant multiple of FC, and therefore still as square of the depth.

From the result of the above demonstration, some authors have attempted to calculate the lateral strain, or strength, from knowing the absolute strength; but this is a problem impossible to be solved, unless we also know the relation of the forces that will crush the body, and that will pull it asunder; for the force whereby any body resists being broken laterally, is compounded of these two forces in an undetermined proportion to the force of each. But the relation of these two forces is so various in different substances, that a set of ex-

periments on every different substance would be required to ascertain it; for there are some bodies that will bear much more as a pillar than as a stretcher, and others quite the reverse.

Cor. 1. In square beams, the lateral strengths are in proportion to the cubes of the sides.

Cor. 2. The lateral strengths of similar homogeneous bodies are in proportion to the cubes of the similar sides of the section.

PROPOSITION LI.

The lateral strength of any beam, or bar, projecting from a wall, with one end supported, or fixed into the wall, to overcome a given weight, or strain, is inversely as the distance of that weight from any section acted upon.

Demonstration. Because from the principle of the lever, (fig. 83,) the strength of the section EF is evidently inversely, as the length of the arm FC from the fulcrum F.

Cor. The strain upon any section is directly as the distance of the weight from that section.

PROPOSITION LII.

If a projecting beam AB, (fig. 84,) be fixed into a wall, and a weight P suspended from the other

PLATE V.

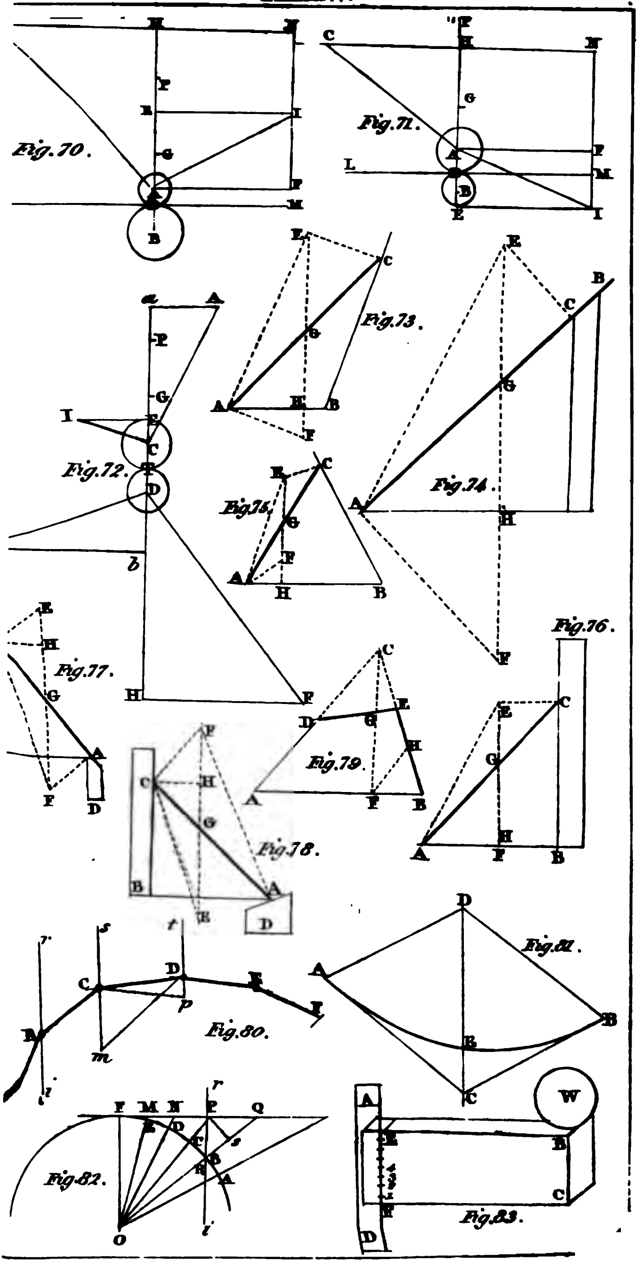


Fig. 70.
Fig. 71.
Fig. 72.
Fig. 73.
Fig. 74.
Fig. 75.
Fig. 76.
Fig. 77.
Fig. 78.
Fig. 79.
Fig. 80.
Fig. 81.
Fig. 82.
Fig. 83.

end, the strain at A by the weight P is the same as the strain upon the middle of a beam of twice the length, with twice the weight P laid upon its middle; this beam being supported at both ends.

Demonstration. Let BA be produced to an equal length beyond A, and a weight equal to P hung at the other end. Suppose the point A to rest upon a fulcrum, then the lever will be in equilibrio. But the weight now on A, or pressure downward, is double the weight P; and the lever is now double the length of AB, and the strain upon the point A is evidently the same as if fixed into the wall.

Cor. The strength of a projecting beam is half the strength of a similar beam, of the same length, with both ends supported.

PROPOSITION LIII.

If a weight lie upon any point C, (fig. 85,) of a beam AB, supported at both ends, the strain at C is in proportion to the rectangle $AC \times CB$.

Demonstration. For we may suppose the lever AB to rest on C as a fulcrum. Now the weight at B to balance the lever is as AC, and the weight at A is as BC, therefore the weight at B and A together is as $AC \times BC$; and the weight at A and B together is equal to the weight or strain at C.

Cor. 1. The greatest strain of a beam is when the weight lies in the middle, the beam being supported at both ends.

Cor. 2. The strain upon any beam at any point P, (fig. 86,) by a weight upon any other point C, is in proportion to the rectangle $AP \times CB$.

Demonstration. Because the strain or action at C is as $AC \times CB$, (Prop. LIII.) and by the principles of the lever $AC : AP :: \text{action at } C = AC \times CB : \text{action of } C \text{ at } P = \frac{AC \times CB \times AP}{AC} = CB \times AP$.

Cor. 3. The strain at C, (fig. 86,) by the weight at P, is the same as the strain P by the weight at C.

PROPOSITION XLIV.

If a weight be equally diffused over a beam AB, supported at both ends, the strain at any point C is one half the strain at C, with the whole weight laid on that point.

Demonstration. Let that part of the weight diffused over CB (fig. 86,) be collected into the centre of gravity D, in the centre between C and B, its action on C and B will remain the same as before, viz. there will be one-half the weight on each. Let the weight between A and C be collected also into

the point E, the centre between A and C; its action on C will remain the same as before. But by the principles of the lever, the action on C, by the weight at D, is one-half of what it would be with the weight at C; and the action at C of the weight at E is one-half of what it would be with the weight at C, and the sum of these actions is the strain at C.

Cor. 1. The strain at C, (fig. 87,) with the whole weight placed there : is to the strain C, with the same weight equally diffused between C and P : : AC : AP$+\frac{1}{2}$PC.

Demonstration. Let the weight pressing equally between C and P, be collected into the centre F, between P and C; then its action on P and C will not be altered, and the action on C, the weight being placed there : is to the action on F : : AC : AP$+\frac{1}{2}$PC. But the action at F, with the weight at C, is the same as the action at C, with the weight at F.

Cor. 2. The strain at C, by a weight diffused equally along AP : is to the strain at C by a weight placed on C : : $\frac{1}{2}$AP : AC.

PROPOSITION LV.

If beams bear weights in proportion to their lengths, either similarly situated, or equally dis-

posed over their surfaces, the strain upon each will be in proportion to the square of its length.

Demonstration. Because the strains are as the lengths and weights, and the weights are as the lengths; that is, the strains are as the squares of the lengths.

Cor. The strain of roofing and joisting depends upon this case.

PROPOSITION LVI.

If the ends of a beam be produced beyond the supports A and B, (fig. 85,) and firmly fixed at the ends, it will bear twice as much weight as if laid loosely upon the supports.

Demonstration. Let twice the weight be laid on C, that the beam would carry laid loosely over the props. Now, since the beams AC and BC are fixed at A and B, they may be considered as two separate prominent beams; and together will bear half the weight laid on C, while the whole beam AB bears the other half.

Cor. 1. In this case the strain is equal in every point between A and B, the props.

Cor. 2. This shows the utility of building the ends of joists firmly into the wall to strengthen

them, if the walls are strong; but, if otherwise, there is a very great lever power to shake the walls.

PROPOSITION LVII.

To make a beam, (fig. 88, 89,) lying loosely over two props, equally able in all its parts to carry a weight, laid upon any point C, taken at random, or uniformly diffused over the whole length; the breadth multiplied by the square of the depth, at the given point C, must be in proportion to AC multiplied by BC. Therefore,

Cor. 1. If the sides are parallel vertical planes, (fig. 88,) the square of the depth, or CD^2, must be in proportion to AC multiplied by CB.

Cor. 2. If the upper and under surfaces be parallel, (fig. 89,) the breadth must be in proportion to AC multiplied by CB.

Cor. 3. If the sections are similar figures, (fig. 88,) the cube of the side must be in proportion to $AC \times CB$.

Cor. 4. If the beam is necessarily loaded on a given point C, then, that the beam may be equally strong throughout to resist that strain, we must make the breadth multiplied by the square of the depth in every section between C and either end,

in proportion to its distance from that end. Therefore,

Cor. 5. If the sides are parallel vertical planes, (fig. 88,) we must make $CD^2 : EF^2 :: AC : AE$.

Cor. 6. If the sections are similar figures, (fig. 88,) $CD^3 : EF^3 :: AC : AE$.

Cor. 7. If the upper and under surfaces are parallel, (fig. 90,) the breadth at C : breadth at E :: AC : AE.

Cor. 8. In a prominent beam with one end fixed, the strength of the section should be in proportion to its distance from the weight.*

PROPOSITION LVIII.

In any beam AC, (fig. 13,) standing aslope, and bearing a weight, the strength or strain of that beam AC is equal to the strength or strain of a beam AB of the same thickness, and bearing the

* This theory supposes the materials to be insuperably strong to resist all strains except the transverse, and therefore that the ends of the beams farthest from the fulcrum may be mere points. But in practice this is not the case, for unless there be a certain thickness of material the ends will be either cut or crushed.

same weight, whose length is the base of the right-angled triangle ABC.

Demonstration. Because the pressure upon the plane AC (Prop. XXI. Cor. 2.) is as the base AB, (fig. 13.)

Cor. 1. If any number of beams of equal thickness (fig. 13,) lean against the same perpendicular wall CB, from the same point A, they will all bear equal weights, whatever be their angles of inclination, or of whatever length they may be.

Cor. 2. If the weights, and lengths of the beams be the same, their strains will be in proportion to the distances of their lower end A from the perpendicular BC, (fig. 13,) or as the cosines of their elevations.

Cor. 3. All that has been said of horizontal beams is equally true of sloping beams, by making the strains in proportion to the base of the right-angled triangle, of which they are the hypothenuse, or as the cosines of their elevations.

All that has been demonstrated in the whole of the above theory, is equally true for any kind of forces whatever, instead of a weight, and in whatever directions, besides that of gravity.

cast iron pillars that support the galleries of some churches solid instead of hollow cylinders.

So far as I have advanced in the theory of transverse strength and strain, I agree with former writers upon the subject, and also with experiments, which have confirmed the above theory.

I now beg leave to depart from the most of former writers upon one part of the theory. It is asserted by eminent authors, that a triangular beam will bear twice as much, or (by some) thrice as much, with the weight or strain placed upon its vertex, as when placed upon its base; and they demonstrate this from the celebrated proposition of Galileo, which is Prop. L. of this book nearly. It is there supposed that the whole of the fibres are torn asunder; and in breaking the whole section of fracture opens, by turning round the point or line in contact with the straining force, as on an immoveable fulcrum or joint.

Now, (as I have there remarked,) this is not the case: it depends wholly upon the nature of the substance employed how far this fulcrum or neutral point may be removed towards the opposite side; for it is obvious, that, if a body be easier crushed than torn asunder, this fulcrum, in beams whose sections are parallelograms, will be nearest the side that inclines to be convex; that is, where the force of cohesion and the force opposed to crushing may balance one another. This circumstance, however, cannot alter any part of the pre-

ceding theory, as far as regards bodies of the same material, whose sections are parallelograms; because, wherever the fulcrum is situated in one body, it must be similarly situated in others of the same substance and position, whichever of the opposite sides be uppermost.

But in a triangular beam, it is evident, that, if the force of cohesion be equal to the force opposed to crushing, the fulcrum or neutral point will accommodate itself to a situation where the forces will balance one another, whatever side be in contact with the straining force, and will be equally strong the one way as the other. But, if the force of cohesion be greater than the force opposed to crushing, then the triangular beam will be strongest with the edge in contact with the strain, but, if less, the contrary.

Now it appears, from the following experiments, that the cohesion of all sorts of timber is greater than its resistance to crushing; and therefore,

PROPOSITION LXII.

A triangular beam of timber is stronger with its base in contact with the straining force than with its vertex; and beams of timber whose sections are trapezoids are strongest with the broader of the parallel sides applied to such strain.

I am persuaded that the above proposition would be disputed by many, if it were not that it is easy to try the experiment.

The following are experiments that have been made upon the transverse strength of iron and timber. The experiments upon the different kinds of timber and metal appear to be few on this kind of strain; but oak, fir, and cast iron, are generally employed for this purpose; if other substances are so employed in taking the ratios of the absolute strengths given there in the tables, there can be little error.

From the numerous experiments of M. Buffon, on a large scale, by direction of the French government, on the lateral strength of oak, he found, that all his experiments agreed with the theory very nearly, with a small deficiency from it in the longer pieces, which may be accounted for by their own weight not being considered. After reducing all his measures to the English standard, he found that a piece of oak 10 feet long, and 4 inches square, broke under the weight of 4015 lb., which is the mean result of a vast number of experiments, which it is unnecessary here to detail.

A bar of American fir, 1 inch square, laid over two props, exactly 1 foot apart, I found, by a variety of experiments, to break with a weight of 206 pounds hung upon its middle, being the average of the experiments. Also, the result of those experiments proved, that the strength of the pieces were in proportion to the square of the depth, multiplied by the breadth, and divided by the length, and that the minute variations from this theory

might be accounted for from circumstances attending the particular cases. I also found, that in the same manner a bar of Memel fir broke in a short time, being loaded with 390 lb. at a mean.

I procured two bars of Norway white batten, cut from the same piece, both free of knots or blemishes, and both exactly equilateral triangles of four-tenths of an inch on the side, placed one of them upon two props, exactly one foot apart, with an edge uppermost, and it broke in a short time with 7 lb. 1 oz. suspended in a scale at its middle. I placed the other upon the same props, with a side uppermost, and it sustained 7 lb. 12 oz. before it broke. They both broke with an oblique fracture about 2 inches long.

I then placed the props at 6 inches apart, and hung weights to the two longest of the broken pieces in the same manner, but reversed them, by turning up the edge of that which was formerly down, and it broke with 11¾ lb. The other, having the side up that was formerly down, broke with 12 lb. The fractures were similar to the others formerly broken.

I procured two bars of American white pine from the same piece, whose sections were trapezoids, their parallel sides were 42 and 18 hundred parts of an inch respectively, and 27 hundred parts between the sides, exactly equal in shape and quality, and placed one with its narrow side uppermost, over two props 11¼ inches apart; it broke

with $14\frac{3}{4}$ lb. suspended upon its middle; and the other, with its broad side uppermost, broke in the same manner with $15\frac{3}{4}$ lb. The fracture of the first was nearly straight across, and of the last, one of the arms was split all along. These experiments prove what is said in Proposition LXII.

The Experiments of Mr Banks upon Cast Iron.

"The bars were one inch square, and the props exactly a yard distant; one yard in length weighed exactly 9 lb.: they all bent about one inch before they broke.

"1 bar broke with	963 lb.	Mean $971\frac{2}{3}$ lb.
"2 bar broke with	958	
"3 bar broke with	994	
"Bar equally thick in the middle, but the ends formed into a parabola, and weighing 6 lb. 3 oz. broke with	864."	

This gentleman made many other experiments, and concludes that cast iron is from $3\frac{1}{2}$ to $4\frac{1}{2}$ times stronger than oak, and from 5 to $6\frac{1}{2}$ times stronger than deal; but the kind of deal is not mentioned.

The Experiments of Mr Rennie on Cast Iron.

"A bar of cast iron 32 inches long, 1 inch square, and 9½ lb. weight, resting upon horizontal props at its ends, bore 1086 lb. at its middle; but half the length supported 2320 lb.

"A bar 2 inches deep, and ½ inch thick, sustained in the same manner 2186 lb.

"A bar having a depth of 3 inches, and ⅓ inch thick, supported 3588 lb.

"An equilateral triangle, of 1 inch in area of section, when resting upon its angle, bore 840 lb.; but when resting upon its base 1437 lb."

This last experiment proves, that the force opposed to crushing is much greater than the absolute cohesion of cast iron; and, by comparing the experiments of Mr Rennie for the absolute cohesion, and those given at end of the strength opposed to crushing, it will be seen, that the former is more than eight times the latter. I mention this to show, that it confirms the reasoning on which Proposition LXII. is founded. The whole of the experiments of Mr Rennie prove the truth of the theory.

There is a visible discrepancy between the weights of Mr Banks' bars and those of Mr Rennie; the former being 3 feet, and only 9 lb. and the latter 32 inches and 9½ lb. Now, 3 feet at 1 inch square of cast iron, according to all the tables of

specific gravity that I have seen, should be 9⅜ lb.; the other results exactly agree. We shall therefore assume them as a standard for the lateral strength of cast iron, the results of M. Buffon's experiments for oak, and beg leave to insert my own for fir, for want of others. We then obtain the following table of the lateral strength of these substances, assuming the standard to be a bar 1 foot long and 1 inch square; which is most convenient for future calculations.

TABLE

Of the Lateral Strength of the following Materials, the bar being 1 Foot long and 1 Inch square, of good quality:

	Weight that will break them.	Weight they can bear with safety.
Cast iron . .	3270 lb.	1090 lb.
Oak . . .	627	209
Memel fir . .	390	130
American white pine	206	69

One-third of the weight that will break them is here assumed for the weight they can bear with safety; and even then it must be of good quality.

Rule. To find the weight that any beam or bar of the above materials will bear with safety on its middle when loosely supported at the ends.

Multiply the square of the depth by the breadth

in inches, and this product again by the tabular number, and divide by the length in feet: the quotient is the answer.

EXAMPLE I.

What weight will a cast-iron bar bear with safety, 10 feet long, 10 inches deep, and 2 inches thick, laid on edge?

$\frac{10^2 \times 2 \times 1090}{10} = 21800$ lb. Ans.

EXAMPLE II.

What will the same bar bear laid on its broad side?

$\frac{2^2 \times 10 \times 1090}{10} = 4360$ lb. Ans.

Having the length and depth of a given beam or bar; to find the breadth required to sustain a given weight.

Multiply the length by the weight, and divide by the product of the tabular number, multiplied by the square of the depth: the quotient is the answer.

EXAMPLE.

An oak beam 20 feet long, and 14 inches deep, is required to sustain 10000 lb.; what must be its breadth?

$\frac{20 \times 10000}{14^2 \times 208} = \frac{200000}{40964} = 4{\cdot}85$ inches the breadth required.

Having the breadth and depth of a beam or bar; to find the greatest length that is sufficient to sustain a given weight.

Multiply the square of the depth, the breadth, and tabular number together, and divide by the weight: the quotient is the answer.

EXAMPLE I.

A Memel beam is 12 inches deep, and 4 broad; required the greatest length that may sustain 5000 lb.

$$\frac{12^2 \times 4 \times 130}{5000} = 12{\cdot}77 \text{ feet length required.}$$

EXAMPLE II.

Required the length, the beam being laid upon its broad side.

$$\frac{4^2 \times 12 \times 130}{5000} = 4.99 \text{ feet, Ans.}$$

Having the length and breadth of a beam or bar, and the weight it is required to sustain; to find its depth.

Multiply the length and weight together, divide by the product of the breadth and tabular number; and the square root of the quotient is the answer.

EXAMPLE.

A beam of American fir is 20 feet long, and 4 inches broad, it is required to sustain a weight of 2000 lb.; required the sufficient depth.

$$\sqrt{\frac{2000\times 20}{69\times 4}} = \sqrt{145} = 12+\text{inches, Ans.}$$

Investigation of the Rules.

Let L = the required length, l = tabular length = 1.
B = the breadth, b = tabular breadth = 1.
D = the depth, d = tabular depth = 1.
W = the weight,
w = the tabular number.

Then $\frac{D^2B}{LW}=\frac{d^2b}{lw}=\frac{1}{w}$; and therefore $W=\frac{D^2Bw}{L}$, $B=\frac{LW}{D^2w}$, $L=\frac{D^2Bw}{w}$, and $D=\sqrt{\frac{LW}{Bw}}$.

PRACTICAL REMARKS.

It must be recollected, that half the weight of the beam must be added to the weights in the foregoing calculations; for half the weight of the beam exerts its force at the middle.

The strength of bodies is materially influenced by their own weight; they may indeed be so long that they cannot support it alone.

From what has been demonstrated, it is evident, that the strength of similar solids does not increase in proportion to their weight; for the strength of similar beams in similar sections is as the cubes of

their corresponding dimensions; but the weights of the bodies are as the cubes of their corresponding dimensions, and therefore the strain upon their sections by their weight is as the fourth powers of their dimensions. Whence a beam of any material may be so large, that the strain by its own weight may far exceed its strength, while a less beam, similar and similarly situated, may support a great weight.

These considerations show, that in all machines where the strain is effected by the weight of the parts, the small bodies are more able to withstand it than the greater. That in two similar engines or machines of different sizes, or such that the quantity of materials of the same kind is in proportion to the size, the less is more efficient in proportion than the greater, and is more able to stand the strain of its own weight. Therefore, when machines are diminished, their strength is not diminished in the same proportion; and a thing may look very strong in a model, that will upon a large scale fall asunder by its own weight.

That there is a boundary set by nature to the size of all machines, constructed of any given materials, which art can never go beyond; for we can calculate the size of any beam, lever, &c. of whatever given materials, shape, or situation, that would fall asunder by its own weight, and therefore no machine, ship, structure, or building whatever, can reach that limit. Thus we see a tender plant

could not be the size of an oak, nor a soft insect the size of a man. That the strength and agility of small animals is much superior to large ones. A man by falling his own height may break his firmest bones; a cat may fall much higher, and not be hurt, and an ant from the top of a steeple in perfect safety. Some insects can leap 500 times their own length, while a man could not raise his body from the ground on limbs of the same construction.

PROPOSITION LXIII.

Problem. Given the length AB, (fig. 93,) the weight w, of a beam supporting a given weight P, which it is just able to bear; to find the length of a similar beam EF, similarly situated, and of the same kind of material that is just able to support its own weight.

Rule. Multiply twice the length of the smaller beam by the weight it supports, and divide the product by its own weight; to the quotient add its own length; and the sum is the length of a similar beam, that can only support its own weight.

Demonstration. The strength of the sections AC and DE are as AB^3 and EF^3, and the weights of the beams are as AB^3 and EF^3. Now AB^3 : $EF^3 :: w : \frac{EF^3 \times w}{AB^3} =$ weight of EF, and the strain on the section $\overline{\tfrac{1}{2}w+P\times AB}$; also the strain on the

section DE is $\frac{w \times EF^3}{AB^3} \times \frac{1}{2}EF$, and the strengths must be in proportion to the strains; therefore $\overline{\frac{1}{2}w+P} \times AB : \frac{w \times EF^3}{AB^3} \times \frac{1}{2}EF :: AB^3 : EF^3$, or $\overline{\frac{1}{2}w+P} \times AB \times EF^3 = \frac{EF^3 \times w}{AB^3} \times \frac{1}{2}EF^3 \times AB^3$, which equation reduced gives $EF = \frac{2AB \times P}{w}$.

Cor. The same rule is true in whatever position the beams are placed, if similarly.

PRACTICAL REMARKS ON JOISTING AND ROOFING.

From the foregoing propositions it is easy to show, that the strength of joisting ought to be in proportion to the squares of their lengths, or, if supported beneath, their strength ought to be in proportion to the squares of the distances of the supports. For we may suppose that all apartments are meant to contain a quantity of furniture in proportion to their size, or otherwise to accommodate occasionally a number of persons in that proportion; wherefore they support a weight or strain in proportion to the length of the joist, and the strain with equal

weights is also as that length; that is, the joint strain is in proportion to the square of the length by Prop. LV. For instance, if a set of joisting has twice the length of another set between the supports, the strength of the first in any section should be four times the strength of the other in a similar section. This is an important truth beyond all controversy, and yet it does not seem to be generally known or attended to by builders. Suppose it to be known what size of the section of a joist is sufficient for a given length in a certain case, let it be required to find the section of a similar joist of a different given length in a similar case.

Rule. Multiply the cube of the depth of the joist whose section is known, by the square of the length of the joist whose section is required, and divide the product by the square of the length of the known joist; the cube root of the quotient is the depth of the section required.*

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* Let L represent the length of the known joist, and D its depth; let l represent the length of the joist whose section is required, and d its depth. Then, because the lateral strength of similar joists is as the cubes of their depths directly, (Prop. L. Cor. 2,) and as the squares of their lengths inversely, (Prop. LV.) it follows that where the strengths are equal, $D^3 l^2 = d^3 L^2$, or $d = \sqrt[3]{\frac{D^3 l^2}{L^2}}$.

Add three times the logarithm of D to twice the logarithm

EXAMPLE.

If in a certain case a joist whose depth is 1 foot, and thickness 3 inches, be sufficient for a length of 30 feet; what must the section of a similar joist be in a similar case whose length is 15 feet? By the rule the depth $= \sqrt[3]{\frac{1^2 \times 15^2}{30^2}} = \sqrt[3]{\frac{225}{900}} = \sqrt[3]{.25} =$ .6298 feet, and 1 feet : 3 inches : : .6298 feet : 1·8894 inches = thickness of the similar beam.

It will be found, by multiplying the thickness by the square of the depth in each of these beams, that the section of the one is four times the strength of the other, as it ought to be. But the areas of the sections are as 3 to 1·19, and as the less is half the length of the greater, the quantity of material in the one is therefore above five times as much as in the other; and yet, although it may seem a paradox, the one has been shown to be as strong as the other.

The above rule may be extended to dissimilar beams.

Suppose we know, as above, the three dimensions of a beam that is of a sufficient strength in a certain case. Let it be required to find any one dimension of any beam that will be equally strong with the given beam in a similar case.

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of l, from the sum subtract twice the logarithm of L, and the remainder divided by 3 is the logarithm of the depth d.

To find the length, the depth and thickness being given.

Rule. Multiply the square of the depth of the unknown beam by its thickness; multiply this product again by the square of the length of the known beam; divide the product by the product of the square of the depth and the thickness of the known beam, and the square root of the quotient is the length required.

EXAMPLE.

Suppose a joist 30 feet long, 12 inches deep, and 3 inches thick, has sufficient strength in a given case, it is required to know the length of another joist of equal strength, whose depth and thickness are 8 and 6 inches respectively in a similar case.

By the rule, $\sqrt{\frac{8^2 \times 6 \times 30^2}{12^2 \times 3}} = \sqrt{\frac{64 \times 6 \times 900}{144 \times 3}} = \sqrt{800} = 28{\cdot}28$ feet, Ans.

To find the depth, the length and thickness being given.

Rule. Multiply the square of the depth of the known beam by its thickness, and this product again by the square of the length of the beam whose depth is required; divide the product by the product of the square of the length of the known beam, and the thickness of the other; and

the square root of the quotient is the depth required.

EXAMPLE.

Suppose a joist 30 feet long, 12 inches deep, and 8 thick, be of sufficient strength in a given case; it is required to know the depth of another joist, whose length is 28·28 feet, and thickness 6 inches, of the same strength in a similar case.

By the rule, $\sqrt{\frac{12^2 \times 3 \times 28{\cdot}28^2}{30^2 \times 6}} = \sqrt{\frac{144 \times 3 \times 800}{900 \times 6}} = \sqrt{64}$ $= 8$ inches, Ans.

To find the thickness, the length and depth being given.

Rule. Multiply the square of the depth of the known beam by its thickness, and this again by the square of the length of the beam whose thickness is required; divide the product by the product of the square of the length of the known beam, and the square of the depth of the other, and the quotient is the thickness required.*

* Let L, D, l, d, represent as before, let T and t represent the thickness of the beams, then by Prop. L. Cor. 2, and Prop. LV., when the strengths are equal, $D^2Tl^2 = d^2tL^2$, from which equation we obtain, $l = \sqrt{\frac{d^2tL^2}{D^2T}}$, $d = \sqrt{\frac{D^2Tl^2}{tL^2}}$, and $t = \frac{D^2Tl^2}{d^2L^2}$.

EXAMPLE.

Suppose a joist 30 feet long, 12 inches deep, and 3 inches thick, has strength sufficient for a certain purpose; it is required to know the thickness of another joist of equal strength, whose length is 28·28 feet, and whose depth is 8 inches for a similar purpose.

By the rule, $\frac{12^2 \times 3 \times 28{\cdot}28^2}{8^2 \times 30^2} = \frac{144 \times 3 \times 800}{64 \times 900} = 6$ inches, Ans.

It is evident also, from the foregoing propositions, that a joist is four times stronger when supported in the middle. That the main part of the strength of a joist consists in its depth; for it may be shown, that a joist may have twice the strength of another, and yet have less timber in it of equal quality and length; for instance, suppose a joist to be 6 inches square, its lateral strength in any section is the square of 6 multiplied by 6, equal to 216. Let another joist be 11 deep, and 3 inches thick, its lateral strength is the square of 11 multiplied by 3, equal to 363; this last is above one-third stronger than the first; but, suppose this last joist laid on its broad side, the depth will be 3 inches, and its strength is the square of 3 multiplied by 11, equal to 99, nearly four times weaker than in the other position, while the quantity of timber evidently remains the same. The quantity of timber in the first joist is $6 \times 6 = 36$, and the quantity in the last is $11 \times 3 = 33$. Deep and

thin joists are therefore the strongest when they have no side pressure; they are also the lightest, and the most efficient, for they do not strain the building so much by their own weight, and very slight levers are sufficient to prevent them from warping.

It has also been shown, (Prop. LVI.) that joists are twice as strong with their ends firmly built into the wall, as when the ends are merely laid loosely upon it,—a circumstance that ought certainly to be attended to when the walls are strong; but if not, they in that case have a tendency to shake the walls.

What has been said of joisting is also applicable to roofing, in as far as regards the strength and strain of the scantling in roofs of the same degree of slope. For the weight of the materials used in roofing is in proportion to the length of the scantling; also the action of the wind is in that proportion. Therefore the strength of any section of the scantling ought to be in proportion to the square of its length in roofs of the same degree of slope, and under similar circumstances. So far the theory of roofing agrees with that of joisting.

It may now be shown, that the strength and strain are modified by the different degrees of pitch that roofs may have. For let ABC and ADC (fig. 96) be the rafters or couples of two roofs of equal breadth, but of unequal heights, it has been shown, (Prop. LVII. Cor. 1,) that, if

AB and AD be of equal thicknesses, they are capable of supporting the same weight, notwithstanding their difference of length. But in roofing the weight on AB : is to the weight on AD : : as AB : is to AD; therefore the strength of a section of the scantling should in all roofs of the same breadth be in proportion to the length of the couple-leg. From this it evidently appears, that a low roof is both lighter and cheaper than a higher one of the same efficient strength; that both the weight and the expense of the scantling of roofs are in proportion to the squares of the lengths of the couple-legs; and that both the weight and the expense of the other materials are in proportion to the lengths of the couple-legs simply.

But flat or low roofs have the disadvantage of a tendency to push out the side-walls of the house, if they are not properly bound together at the bottom. To illustrate this, the couples of a roof support one another at top mutually, and they may be supposed the same as if leaning against a wall, with this difference, that the pressure of each couple-leg at top is thrown against the other; it may therefore be deduced from Proposition XLIV., that as the height of the roof above the side-walls : is to its weight : : so is half the width of the house : to the horizontal pressure.

This rule holds strictly true when the centre of gravity of each side is in its middle, as it is generally very near it; but the lower the centre of gra-

vity is, the less is the side-pressure, and therefore an advantage is gained by placing the heaviest of the materials lowest down.

The intelligent mechanic will easily perceive, that the strength of the stretchers, or ceiling-joists, for binding a roof, and also the mode of attaching them, should be such as to bear a weight as a stretcher equal to the calculated side-pressure.

He will also perceive, that the strain upon flanks, when unsupported, beneath is very great, and ought to be avoided as much as possible; but that the strain upon peends and ridges is little or nothing. By combining Prop. XLIV. and XLV., we might calculate the strains upon flanks; but these strains vary by so many different circumstances, that a general rule to answer all cases is almost impossible; individual cases of these may be obtained from the above-mentioned propositions; and complicated or irregular roofs from Prop. XLVII. and its corollaries, on the principles of which, figure 97 is the elevation of a light, though strong wooden bridge of 60 feet span, which, upon the same principles, the intelligent artist may modify at his pleasure; and upon the same principle, he will also see how to construct the centres for stone arches.

Experiments show, that beams opposed to any considerable strain are bent into curves, and that the space bent through is always in proportion to the weight or strain. The degree of deflection,

however, depends upon the internal structure of the material. This is called the elastic curve. All notches or mortices ought to be cut on that side of the beam against which the strain is applied, but never on the other side; for a small incision on the side that inclines to be convex weakens the beam very materially; while, by the experiments of M. Duhamel, a bar of willow was cut one-third of its depth on the side inclining to be concave, and the slit filled up with a slip of hard wood, and the bar was thereby one-sixth stronger than at first. This experiment also shows the importance of filling up all mortices tightly.

All trees are composed of a number of eccentric cylinders, or annual growths, nearly circular. In the northern hemisphere the centres of these annual plates are nearer to the north than the south side of the tree, and the reverse in trees of the southern hemisphere; and the difference is most remarkable in trees growing singly, and exposed to the sun's rays. M. Buffon found, that the south side of the tree was always the strongest timber.

These annual plates are supported together by the air-vessels, containing a substance much weaker than the ligneous fibres, and much easier separated, as may be seen in the splitting of lath. The artist, therefore, in all cases where it is admissible, should cut his timber so that the edges of the annual plates should as much as possible oppose the strain. In large beams this is scarcely possible;

but in small ones, such as the backs of chairs, hammer, pick, and shade shafts, &c. and even in roofing and joisting, it is often practicable. Thus let A and B (fig. 94,) be two battens, cut out of the same tree, to bear a weight upon their upper edges, A will in a manner resemble so many boards laid on edge to support the strain, while B has a like resemblance to the boards laid upon their broadsides.

THE STRENGTH OF MATERIALS TO RESIST BEING CRUSHED.

THIS kind of strain seems to be less understood than any of the others; there have been but few experiments made upon it; for these kinds of experiments are always attended with trouble and expense, and upon a large scale are beyond the ability of private persons.

The celebrated M. Euler, and from him many other authors of high repute, have demonstrated, (without previous experiment,) that the strength of a column to resist this kind of strain is in proportion to the fourth power of the diameter directly, and the square of its length inversely; but in this he took into consideration only the chance of the column being bended, which has nothing to do with the resistance to being crushed. It is reason-

able to allow, that the chance of the bending of a straight column by nothing but a longitudinal strain is altogether a fortuitous circumstance, that can scarcely be brought under calculation; that it cannot bend unless it is acted upon either directly or indirectly by some kind of lateral pressure, and therefore does not come under this description of strains.

In order to discover the law that regulates this kind of strain, so far as regards the area of section and the length of the column, I made a great number of experiments upon dried clay cast into cylinders of various lengths and diameters, and came to a most satisfactory conclusion, as far as regards friable substances, which may extend to all kinds of stone and cast iron, viz.

PROPOSITION LXIV.

The strength of a column is directly as the square of the diameter, or the area of section, let its length be what it may, unless it be so long as to have a chance of bending.

The following is a tabular view of the experiments:—

TABLE

Showing the Weights by which Cylinders of dried Clay of the following Lengths and Diameters were crushed.

Inches.		Pounds.			Inches.		Pounds.		
Length.	Diameter.	Weight.	Number to which they approximate.	Mean.	Length.	Diameter.	Weight.	Number to which they approximate.	Mean.
.25	.25	10.2			1	.7	81		
.5	.25	9.8	10	9.975	1.6	.7	76		
1.	.25	9.6			2.	.7	74	78.4	78
1.5	.25	10.3			3.	.7	83		
.5	.35	18.5			3.75	.7	77		
.75	.35	20.1			4.	.7	79		
1.	.35	19.9	19.6	19.24	1.2	.75	91		
1.6	.35	18.75			2.3	.75	86		
2.1	.35	18.2			3.1	.75	94	90	89.6
3.1	.35	20.			4.	.75	88		
.5	.5	40.2			5	.75	89		
1.	.5	38.5			.5	1	158		
1.4	.5	42.1	40.	39.9	1.5	1	149		
1.75	.5	39.4			2.	1	169	160	160.16
2.75	.5	38.5			3.4	1	153		
4.	.5	40.7			4.5	1	170		
.6	.6	58.5			5.5	1	162		
1.3	.6	59							
2.	.6	56	57.5	57.4					
3.5	.6	57.5							
4.5	.6	56							

In Mr Rennie's experiments, cubes of one-fourth inch were crushed with the following weights :—

	lb.
Iron cast vertically, . .	11140
——— horizontally, . .	10110

	lb.
Cast copper,	7318
Cast tin,	966
Cast lead,	483

Cubes of an inch were crushed by the following:—

Elm,	1284
White deal,	1928
English oak,	3860
Craigleith freestone,	8688

Cubes of 1½ inch side, or 2¼ inch section, by the following:—

Red brick,	1817
Yellow baked brick,	2254
Fine brick,	3864
Craigleith stone with the strata,	15560
——— across the strata,	12346
White statuary marble,	13632
White veined Italian marble,	21738
Purbec limestone,	20610
Cornish granite,	14302
Peterhead granite,	18636
Aberdeen blue granite,	24536

It has been said, by writers who have not tried the experiments, that resistance to crushing in-

creases in a greater ratio than the area of the section. Perhaps it may be so in timber, or other substances of a fibrous texture; but it is not so in stone, nor any substance of a granulous texture, as evidently appears from Mr Rennie's experiments upon Craigleith stone, and my own upon dried clay, which was harder than some stone used in building. The same writers also imagine that the strength decreases with the length. If so, two or more cubes of timber or stone laid above one another must be stronger than a solid piece of the same length and thickness; for they will be as difficult to crush above one another as separately; or that a column of one solid stone is weaker than one composed of a number of stones, which is absurd.

STRENGTH OPPOSED TO TWISTING.

PROPOSITION LXV.

The strength of homogeneous cylinders in being twisted or wrenched round the axis is as the cubes of their diameters. (Fig. 95.)

Demonstration. Let ABC and *abc* be the sections of two cylinders, and let the sections be supposed to be composed of an indefinite number of circular rings in proportion to their diameters, now

the effort of each particle or fibre in each ring to withstand the force of a twist acting at the circumference is as their radii AD to *ad*, by the principles of the lever, or as the diameters, and the number of particles in each ring is as the squares of the diameters; therefore the whole effort to withstand the twist is as the cubes of the diameters.

Cor. The strengths of hollow cylinders to withstand a twist, their quantities of matter being the same, are also as the cubes of their greater diameters.

HYDROSTATICS AND HYDRAULICS.

PROPOSITION LXVI.

MOTION or pressure in a fluid is not propagated in straight line, but equally all around in every direction, whether direct, opposite, lateral, or oblique.

Demonstration. This is a necessary consequence of the peculiar structure of fluid bodies. By constant experience we find, that their constituent particles are moved amongst themselves by an indefinitely small force; and the truth of the proposition is fully verified by the following experiment:—

Take a close vessel A, (fig. 100,) into which

insert as many pipes B, B, B, &c. as you please, all open at both ends, either straight or crooked, inserted into either top, sides, or bottom, and in all manner of directions, having their upper ends in the same horizontal level; then pour any fluid, as water, into any of the pipes until it fills the vessel, and rises to the top of the pipe; the water will rise to the same height also in all the rest of the pipes, which it could not do unless the pressure were propagated equally in every direction of the pipes.

If any of the pipes are very small, the water will rise a little higher in them. This is caused by what is called capillary attraction, and does not strictly belong to the laws of hydrostatics.

Cor. 1. Whence, if water, or any other fluid, be communicated through pipes, between any number of places, whether the pipes be crooked or straight, wide or narrow, it will rise to the same level in all the places, and any fluid surface will rest only in a horizontal position.

Cor. 2. From this property of fluids, houses, towns, or cities, are supplied with water from distant reservoirs. If the reservoir is upon the same level with or very little below the tops of the highest houses, the water is conveyed through pipes to the tops of those houses with no other

PLATE . VI.

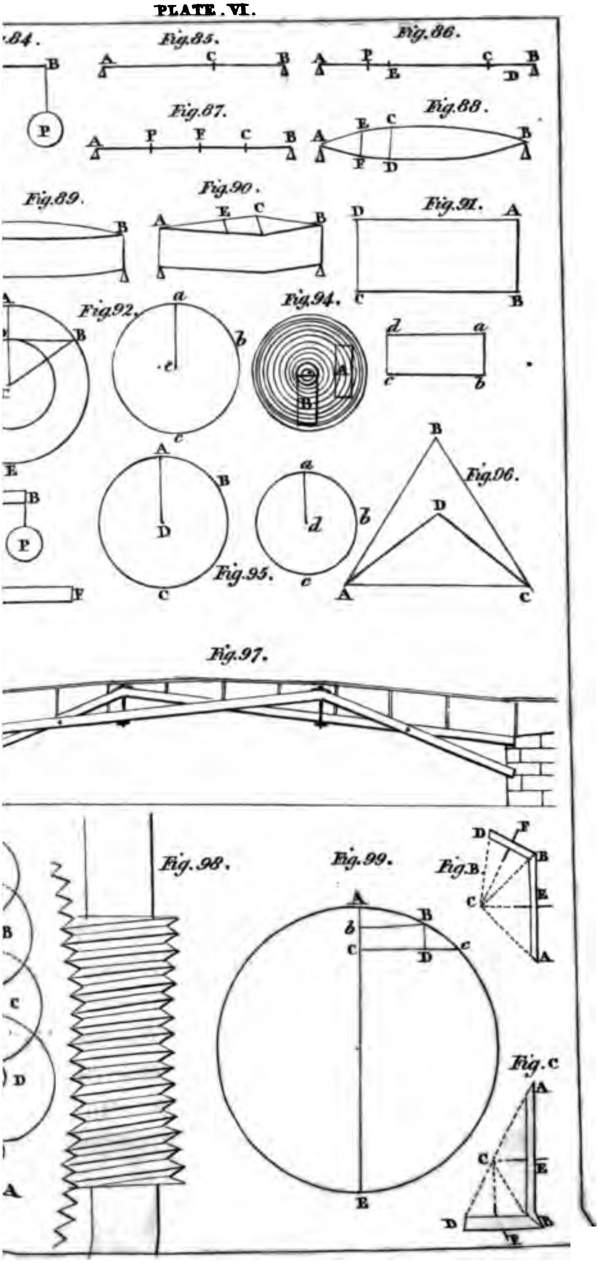
Fig.85.
Fig.86.
Fig.87.
Fig.88.
Fig.89.
Fig.90.
Fig.91.
Fig.92.
Fig.94.
Fig.95.
Fig.96.
Fig.97.
Fig.98.
Fig.99.
Fig.B.
Fig.C

Demonstration. If the vessel be a cylinder or prism, the proposition is evident from the last, (Fig. 101, 102, 103;) but if it be any other figure, as A, B, or C, if a pipe of any area of section, say 1 inch, be inserted into a hole, made in any part of the bottom, and turned upwards, the fluid will rise in the pipe to the same level with that in the vessel. The pressure upon any inch of the base is therefore equal to the height of the column of 1 inch thick; the pressure therefore on all the inches, or the area of the base, is equal to that area by the height of the fluid.

PRACTICAL REMARKS.

The above is called the Hydrostatic Paradox.

LET C be a cylinder of flexible leather, (fig. 103,) firmly attached to two boards DE perfectly air-tight, and about 4 feet in area. Let F be a cylindric pipe of about ½ inch bore, and 5 feet high, inserted tightly into a hole in the board D; if water be poured through the pipe F to fill the leather cylinder, it will lift a weight laid upon the board D equal to the weight of a column of water whose height is 5 feet, and the area of its base 4 feet, equal to 20

cubic feet of water, or 1250 pounds avoir.; and the curiosity is, that all this is effected by a column of water 5 feet high and $\frac{1}{2}$ inch thick, and the same would take place with a pipe no thicker than a straw. By a long and very small pipe, the strongest casks or vessels may be burst asunder by the force of almost the least assignable quantity of water.

For the same reason the retaining walls of roads and bridges are thrust out by the force of the thinnest assignable sheet of water getting between them and the retained earth; whence we see the propriety of having holes at the bottom of such walls to let the water escape.

Cor. 1. The pressure of fluids is directed in everywhere perpendicularly to the surface that retains them (Prop. XX.) with a force equal to the weight of a column of the fluid whose base is that surface, and whose height is the perpendicular to the surface of the fluid.

Cor. 2. The lateral pressure of the thinnest film of the fluid against a perpendicular surface is equal to the pressure of the greatest quantity of the same height against the same surface.

PROPOSITION LXIX.

The quantity of pressure upon any plane surface is the same as the pressure upon the same

N

plane, placed horizontally at the depth of its centre of gravity.

Demonstration. For since the pressure uniformly augments with the depth, from the nature of the centre of gravity, the excess of pressure below it must equal the deficiency above it.

Cor. The centre of pressure of a fluid upon any plane surface, either vertical or inclined, is at the centre of percussion; the surface of the fluid being supposed the centre of motion.

Demonstration. Let 06 (fig. 7,) represent the edge of a rectangular plane sustaining the pressure of a fluid whose surface is at 0; the pressure at 0 is nothing, and is as the depth at any point. Divide the line 06 into any number of equal parts in 1, 2, 3, &c. and draw 1*a*, 2*b*, 3*c*, &c. perpendicular to 06; these lines represent the several pressures at the points 1, 2, 3, &c. and the whole triangle 06A represents the whole pressure upon 06. But the centre of pressure of the triangle is its centre of gravity, (the force being equal upon every point,) that is, at two-thirds the distance from the vertex, which is the same as the centre of percussion of the rectangular plane 06.

PRACTICAL REMARKS.

From the above propositions we discover, that the thickness of the metal or other material in any part of pipes sustaining a fluid should be in proportion to their diameters, multiplied by the perpendicular height of the surface of the fluid above that part.

That the strength of the bottoms of vessels for holding liquors should be in proportion to their areas, multiplied by the depth of the fluids: That the mean strength of the sides should be one-half that of the bottom, and should augment gradually from top to bottom. This agrees also with Prop. LVIII.

That the strengths of sluices, and canal locks are regulated upon the same principles; and the same reasoning may be applied to floating vessels, or those immersed in fluids.

PROPOSITION LXX.

If any body be of the same density with a fluid, it will, when immersed in it, remain suspended in any place. A body with greater density will sink, and a body with less density will float in the fluid.

Demonstration. Let AB be a body immersed in a fluid (fig. 104,) of the same density with itself, then the weight of the space it occupies will

be the same as the weight of the fluid it displaces, and the pressure of the whole column ABCD downward is the same as though no body had been immersed in it; the weight or force therefore pressing up the lower surface is the same as that pressing it downward. The body must therefore remain at rest.

If the weight of the body be greater than an equal bulk of the fluid, the weight of the whole column ABCD is greater than an equal bulk of the fluid; the weight therefore pressing down the lower surface is greater than that pressing it upward. The body must therefore sink.

If the weight of the body be less than an equal bulk of the fluid, in the same manner it may be shown, that the force pressing the lower surface upward is greater than that pressing it downward. The body must therefore rise to the top and float.

Cor. 1. Heavy bodies sink quicker than lighter ones; and, if several bodies be mixed together in a fluid, on subsiding the heaviest will be found at the bottom, those of mean density in the middle, and the lightest on the top; and the same may be said of fluids that are mixed of different densities, if not mixed by chemical affinity. Whence the method of washing insoluble powders or other granular substances.

Cor. 2. The motion of non-compressible bodies,

either sinking or rising in a fluid, will be uniform at all depths, and the heaviest side will always be lowest.

Cor. 3. Bodies immersed and suspended in a fluid lose the weight of an equal bulk of the fluid.

Demonstration. For the whole column ABCD is either equal to the weight of the same bulk of the fluid, or else heavier or lighter than it, only by the difference of the weight of the body AB, and that of an equal bulk of the fluid.

Cor. 4. The fluid acquires the weight that the body loses.

Demonstration. Because the sum of the weights is the same before and after.

Cor. 5. All bodies immersed in a fluid lose weight in proportion to their bulks.

Cor. 6. The weights lost by immersing the same body into different fluids are in proportion to the specific gravities of the fluids.

PROPOSITION LXXI.

The weight of any body floating upon a fluid is equal to the weight of the quantity of the fluid it displaces.

Demonstration. For the pressure of the body downward is the same as the contrary pressure upward; that is, the same as though the body were removed, and the fluid put in its place.

PRACTICAL REMARKS.

From Prop. LXX. Cor. 2, we deduce the theory of an instrument for sounding deep seas. By sinking a strong hollow glass globe by means of a weight attached to it, as soon as the weight strikes the bottom, it disengages itself, and the light vessel rises again to the surface. Now it is evident, that the time it is under water is always proportional to the depth, and the time for a known depth is easily determined by previous experiment. Water will not penetrate glass by any pressure yet known.

The last proposition comprehends the theory of all the instruments for measuring the strength of spirituous liquors.

From it we also discover, that the heaviest ship sailing over an aqueduct has no weight or strain upon the arch whatever; neither does it when floating in a canal lock or basin throw any strain upon any part of them, except only what is made by agitating the water.

From the above propositions we also deduce the following rules :—

TO FIND THE SPECIFIC GRAVITY OF BODIES.

Water is the standard by which the specific gravities of bodies are compared ; its specific gravity being 1. It fortunately happens, that a cubic foot of water at about 60 degrees of Fahrenheit, weighs 1000 oz. avoir. ; so that when we know the specific gravity of any body, by annexing 3 ciphers to it, we have then the number of ounces in a cubic foot of that body. But, in order that the ounces in a cubic foot may be shown in the table at once, we will assume 1000 for the specific gravity of water.

Case 1. *When it is a solid heavier than water.*

Weigh it first carefully in air, and then in water, while completely immersed, and note the weights. Then divide the absolute weight of the solid by the difference of its absolute weight, and its weight in water, the quotient is the specific gravity.

Case 2. *If it be a solid lighter than water,* tie a piece of metal to it, so that the compound may sink in water ; then to the absolute weight of the light solid, add the weight of the metal in water,

and from the sum subtract the weight of the compound in water; divide the absolute weight of the light solid by this difference; and the quotient is the specific gravity of the light solid.

Case 3. *For a fluid.*

Take a solid of known specific gravity that will sink in the fluid; then multiply the difference of the weights of the solid in and out of the fluid by the specific gravity of the solid; divide the product by the absolute weight of the solid; the quotient is the specific gravity of the fluid.

By the above methods, tables of specific gravities have been constructed.

Investigation of the Rules.

Case 1. Let W be the absolute weight of the solid.
$w =$ its weight in water.
$S =$ the specific gravity of the solid.

Then $W-w$ (weight of an equal bulk of water with the solid) : W : : 1 (specific gravity of water) : S; that is, $S = \frac{W}{W-w}$.

Case 2. Let $E =$ the weight of the metal in water.
$F =$ the weight of the compound in water.

Then $F-E =$ the weight of the light solid in water, and the weight of an equal bulk of water is is $W-\overline{F-E}$, or $W+E-F$, wherefore $W+E-F : W : : 1 : S$, or $S = \frac{W}{W+E-F}$.

Case 3. Let s = specific gravity of the fluid; then by Case 1, $wS = \frac{Ws}{W-w}$, (Prop. LXX. Cor. 6,) therefore $s = \frac{W-w}{W}S$.

In the following table of specific gravity, we shall commence first with solids, and then with fluids, beginning with the heaviest, and take them in order down to the lightest.

TABLE OF SPECIFIC GRAVITIES.

Platinum, laminated	22070	Portland stone	2496
——— drawn into wire	21040	Millstone	2484
——— purified	19500	Craigleith stone	2362
Gold pure hammered	19360	Common salt	2130
——— cast	19260	Brick	2000
Lead, cast	11350	Earth	1980
Silver, pure hammered	10510	Ivory	1920
———, cast	10470	Chalk	1790
Bismuth, cast	9820	Coal	1300
Copper-wire	8890		1240
——, cast	8790	Amber	1080
Brass wire	8540	White wax	970
——, cast	8400	Tallow	940
Cobalt	7810	WOODS.	
Steel, soft	7810	Lignum vitæ	1330
Iron, malleable Swedish	7790	Ebony	1180
——— British	7640	Mahogany	1060
——, cast ——	7210	Box and Brazil wood	1030
Pewter	7470	Dry oak	930
Tin, cast	7300	Beech	850
Zinc, cast	7200	Ash	840
Antimony, cast	4950	Plumtree, dry	830
Diamond	3500	Yew	760
Fluor spar	3180	Elm from 800 to	600
White lead	3160	Crab tree	700
Flint glass	2870	Fir from 570 to	500
Jasper	2700	Walnut	650
Rock crystal, slate	2650	Cedar	610
Flint	2590	Cork	240

FLUIDS.			
Mercury	13570	Turpentine	991
Sulphuric acid	1840	Lintseed oil	940
Nitric acid	1220	Olive oil	910
Muriatic acid	1190	Strong alcohol	820
Sea water and milk	1030	Sulphureous acid gas	2.7611
Vinegar	1026	Nitrous gas	1.4544
Tar	1015	Oxygen gas	1.435
		Azotic gas	1.182
Water 60° Fah. distilled, or rain	1000	Atmospheric air	1.2
		Hydrogen gas	0.1

Having the dimensions of any substance given, we can easily find its weight from the table of specific gravities; because it shows the weight in ounces of a cubic foot. As, for example,

What is the weight of a log of mahogany 16 feet long, and 30 inches square?

$16 \times 2.5^2 = 100$ feet. $100 \times 1060 = 106000$ ounces $= 2$ ton 19 cwt. 0 qr. 17 lb.

Required the weight of a bar of Swedish iron 10 feet long, 3 inches broad, and $\frac{1}{2}$ inch thick.

$10 \times 0 \cdot\cdot 3 \times 0 \cdot\cdot 0\frac{1}{2} = 0$ feet 1 in. 3 lines, and 1 f. : 7790 oz. : : 1 in. 3 l. : 60 lb. $13\frac{3}{4}$ oz.

Required the weight of 40 cast-iron bars, each 4 feet long, and $\frac{3}{4}$ inch square.

160 feet $\times \frac{3}{4}^2$ in. $= 7\frac{1}{2}$ in. And as 1 f. : 7210 oz. : : $7\frac{1}{2}$ in. : 4506 oz. $=$ 281 lb. $10\frac{1}{4}$ oz.

In a similar manner the weight of any other substance may be found.

PROBLEM.

In a compound mass of two different substances,

whose specific gravities are known; to find the quantity of each substance.

Rule. Weigh the whole mass, and find its specific gravity by the foregoing rules. Multiply the difference of the specific gravities of the compound and the lighter substance by the specific gravity of the heavier substance, and again by the weight of the compound for a dividend; divide by the difference of the specific gravities of the two substances, multiplied by the specific gravity of the compound; the quotient is the weight of the heavier body, which, being subtracted from the weight of the compound, leaves the weight of the lighter substance.

EXAMPLE.

The weight of a piece of plate, known to be compounded of gold and silver, is 18 ounces troy; its specific gravity is found to be 18700; required the weight of each metal.

18700 specific gravity of the compound.
10470 ——————— of pure silver.

8230 difference.

19260 specific gravity of pure gold.
10470 ——————— of —— silver.

8790 difference.

$$\frac{8230\times19260\times18}{8790\times18700}=\frac{2853176400}{164373000}=17.297 \text{ ounces of gold.}$$

18 compound.
17.297 ounces of gold.
·703 ——— silver.

Investigation of the Rule.

Let H = the weight of the heavier substance, h its specific gravity.

L = the weight of the lighter substance, l its specific gravity.

C = the weight of the compound, c its specific gravity.

Then $H+L=C$, or $L=C-H$, and $\frac{H}{h}+\frac{L}{l}=\frac{C}{c}$. For L substitute its equal C—H, and the equation being reduced, we obtain $\frac{(c-l)h}{(h-l)c}C=H$, and in the same manner $\frac{(h-c)l}{(h-l)c}C=L$.

By reason of chemical affinity, it happens in some cases that, when substances are combined, they contract into less volume than when separate; so that the above rule does not always hold.

1. NEW TABLE

f the Weight of 1 lineal Foot of Swedish Iron, of all Breadths and Thick-nesses, from 1 Tenth of an Inch to 1 Inch, in Pounds and Thousandth Parts.

.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	10ths of Inches.
034	.068	.101	.135	.169	.203	.237	.270	.304	.338	.1
	.135	.203	.270	.338	.406	.473	.541	.608	.676	.2
		.304	.406	.507	.609	.710	.811	.913	1.014	.3
			.541	.676	.811	.947	1.082	1.217	1.352	.4
				.845	1.014	1.183	1.352	1.521	1.690	.5
					1.217	1.420	1.623	1.826	2.029	.6
						1.657	1.893	2.130	2.367	.7
							2.164	2.434	2.657	.8
								2.739	3.043	.9
									3.381	1.0

2. NEW TABLE

f the Weight of 1 lineal Foot of Swedish Iron, of all Breadths and Thick-nesses, from 1 Inch to 6 Inches, in Pounds and Hundredth Parts.

1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	6	In.
[illegible]	4.23	5.07	5.91	6.76	8.45	10.14	11.83	13.52	16.91	20.29	1
	5.29	6.34	7.40	8.45	10.56	12.68	14.79	16.91	21.13	25.36	$1\frac{1}{4}$
		7.60	8.87	10.14	12.67	15.21	17.75	20.29	25.36	30.43	$1\frac{1}{2}$
			10.35	11.83	14.78	17.75	20.71	23.67	29.58	35.50	$1\frac{3}{4}$
				13.52	16.91	20.29	23.67	27.05	33.81	40.51	2
					21.13	25.36	29.58	33.81	42.26	50.72	$2\frac{1}{2}$
						30.43	35.50	40.57	50.72	60.86	3
							41.42	47.34	59.16	71.00	$3\frac{1}{2}$
								54.10	67.62	81.14	4
									84.52	101.44	5
										121.72	6

3. NEW TABLE

Of the Weight of 1 superficial Foot of Swedish Iron-Plate from 100th Part of an Inch thick to 1 Inch.

Thickness 100th Parts of an Inch.	Weight Pounds and 1000th Parts.	Thickness 10th Part of an Inch.	Weight Pound and 1000th Parts.
.01	.406	.10	4.057
.02	.811	.2	8.114
.03	1.217	.3	12.172
.04	1.623	.4	16.232
.05	2.029	.5	20.286
.06	2.434	.6	24.344
.07	2.840	.7	28.401
.08	3.246	.8	32.458
.09	3.651	.9	36.516
.10	4.057	1.	40.573

4. NEW TABLE OF MULTIPLIERS,

For the other Metals, whereby their Weights may be found from the above Tables.

Metals.	Multipliers.	Metals.	Multipliers.
Platinum, laminated	2.846	Copper, cast	1.128
———, purified	2.503	Brass wire	1.096
Pure gold, hammered	2.486	———, cast	1.080
———, cast	2.472	Steel	1.003
Lead	1.457	Iron, Swedish	1.
Pure silver, hammered	1.350	———, British	.980
———, cast	1.344	———, cast	.925
Copper, wire	1.136	Pewter	.960
———, hammered	1.132	Tin, cast	.937

USE OF THE TABLES.

When the weight of 1 lineal foot of Swedish iron bar of any size is wanted, for instance 9 tenths by 7 tenths, then in Table 1, under .9, and in the horizontal line with .7, you have 2.130 pounds, the weight required. In the same manner the weight is found in Table 2.

When the weight of 1 square foot of Swedish plate or sheet iron is required. In Table 3, opposite the thickness, is the weight.

When the weight of any other metal is required. Find the weight of a similar piece of Swedish iron, and multiply it by the number opposite the metal in Table 4.

When the diameter of a round piece of metal is given, find its weight as if square, and multiply it by .7854.

When the piece of metal is not found exactly in the Tables, proportional parts must be taken for the difference; but this will be illustrated by the following

EXAMPLES.

1. Required the weight of a bar of Swedish iron 12 feet long, 3 inches broad, and 1.1 inches thick.

In Table 2. 3.38 (1 by 1 inch) × 3 inches × 12 feet = 121.68 lb.

In Table 1. .34 (1. by .1) × 3 inches × 12 feet = 12.17 lb.

Answer, 133.85

2. Required the weight of an equal bar of cast iron.

133.85 × .925 (Table 4,) = 123.81 lb., Answer.

3. Required the weight of a lead cistern 3 feet square, and 2 feet deep, open at top, the lead being .2 thick.

$3 \times 3 = 9$ feet the bottom.
$12 \times 2 = 24$ the sides.

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33 × 8.114 (Table 3,) × 1.457 (Table 4,) = 390.22 lb., Answer.

4. Required the weight of a circular plate of cast iron 7 feet diameter, and 1.73 inches thick.

By Table 3.	1 foot at	1	inch	= 40.573
	1	at .7		= 28.401
	1	at .03		= 1.217
	1	at 1.73		= 70.191

70.191 × 49(7^2) × .925 (Table 4,) × .7854 = 1673 lb., Ans.

PROPOSITION LXXII.

When a body floats upon a fluid, (fig. 105,) it will rest only when the line Gg, joining the centre of gravity of the body and the centre of

gravity of the displaced fluid, is in a vertical position.

Demonstration. Because the centre of gravity of the body is the centre of all the forces pressing it downward; and the centre of gravity of the fluid is the centre of all the forces pressing it upward; and those forces when the body is at rest are equal and opposite. But the force of gravity is vertical; therefore the line G*g* is vertical.

Cor. The body can be permanently at rest only when its centre of gravity is in its lowest situation. Whence the reason of ballasting ships, and of stowing the heaviest of the cargo lowest down.

PROPOSITION LXXIII.

The quantity of water that flows in a river, canal, or brook, or that is discharged through any pipe in a given time, is in proportion to the area of the section of the stream, multiplied by its velocity at that section.

Demonstration. Because the quantity in any given length is as the section; and the number of those lengths that flow, or that are discharged in a given time, is as the velocity.

Cor. The velocity in any place is in proportion to the area of the section of the stream inversely.

Whence to measure the quantity of water that flows in a river, &c. measure the area of its section in any convenient place, which multiply by the space that a floating body passes over in a given time at that place; the product is the quantity that flows in that time.

PROPOSITION LXXIV.

When water issues through an aperture in the bottom or side of a vessel, its velocity is equal to that acquired by a body falling through a space equal to the height of the surface of the water above the aperture.

Demonstration. For it is impelled by a force that would raise it to that height in a pipe, (Prop. XLVI.) This truth has, however, been satisfactorily established by experiment, with a small allowance of course for the resistance of the air, and the resistance of the sides of the aperture.

Cor. 1. If water spout out of a hole perpendicularly upward, the jet will rise to the height of the fountain-head.

Cor. 2. The velocities, and consequently the quantities of water, flowing through apertures at different heights of the fountain-head, are in proportion to the square root of the heights of the surface of the water above the centres of the

apertures. (Prop. XV. Cor. 1. and Prop. LXXIII.)

Cor. 3. The quantity of water that flows out of a vertical rectangular aperture that reaches as high as the surface, is $\frac{2}{3}$ of the quantity that would flow out of the same aperture placed horizontally at the depth of its base.

Demonstration. Because the quantity flowing out at any depth is as the square root of that depth, or as the ordinate of a parabola of the same base and height. The whole quantity therefore is the sum of all the ordinates, or the area of the parabola, whose base and height are equal to these of the rectangle, and the parabola is $\frac{2}{3}$ of the rectangle.

Experiments show, that when water issues out of a circular aperture, pierced in a thin plate in the bottom or side of a vessel or reservoir, that the stream is contracted into a smaller diameter to a certain distance from the aperture. That the vein is smallest at the distance of half the diameter of the aperture, where the area of its section is to that of the aperture as .62 to 1, or as 10 to 16. That the stream then acquires the velocity given in the theory, and consequently the theoretical discharge of the water must be multiplied by .62 to give the true discharge. That when the water issues through a short tube inserted into the aperture, it

will be less contracted, namely, in the ratio of 10 to 13. And when it issues through a conic frustum, whose greater base is the aperture, the less base at the distance of half the diameter of the aperture, and its area to that of the greater base as .92 to 1, there will be no contraction of the vein. This last form is therefore the best, when a supply of water is agreed upon to flow through a certain given aperture.

PROPOSITION LXXV.

The resistance that a body encounters in moving through a fluid is in proportion to the square of the velocity.

Demonstration. Because the resistance is as the number of particles striking, which number is as the velocity, and as the force by which they strike, which is also as the velocity.

Cor. 1. The resistance that any plane surface encounters in moving through a fluid with any velocity, is equal to the weight of a column of the fluid, whose height is the space a body would fall through to acquire that velocity, and whose base is the surface of the plane.

This follows from the last proposition.

Cor. 2. It is the same thing whether the plane

moves against the fluid, or the fluid against the plane.

Cor. 3. The force of water against the floats of a wheel, is equal to a column of water, whose base is the section of the stream in that place, multiplied by the height of the water to the surface.

Cor. 4. When the quantity of water is given, its force against the floats of a wheel is in proportion to its velocity simply, or to the square root of the height of the surface.

Demonstration. Because the number of particles striking is the same whatever be the velocity; the force therefore is as the velocity, or the section of the stream is always diminished as the velocity increases.

The above holds true only when the water escapes from the wheel, immediately after impinging upon the floats.

From the above propositions, we may calculate the power of a waterfall to drive a mill. In the case of undershot wheels, the power of the fall is in proportion to the area of the section of the stream where it impinges upon the floats, multiplied by the height of the fall; so that if we know the quantity and height of a fall that drives a

known mill, performing a certain quantity of work, we can find what quantity of work any other fall will be capable of performing under similar circumstances.

To find the velocity of the water acting upon the wheel.

Multiply the height of the fall by 64.38, and extract the square root of the product: the root is the velocity in feet per second.

To find the area of the section of the stream.

Divide the number of feet flowing in 1 second, (Prop. LXXIII. Cor.) by the velocity in feet per second; the quotient is the section of the stream in feet.

The section of the stream being thus reduced to its ultimate area, where it acts upon the wheel, (namely, in the inverse ratio of its velocity,) its force upon the wheel will now be in proportion to the square of its velocity, or height of the fall; or its force will be equal to the weight of a column of water, whose base is the section of the stream where it acts upon the wheel, and whose height is the height of the fall. Therefore,

To calculate the power of the fall.

Multiply the area of the section of the stream where it acts upon the wheel by the height of the fall both in feet. This product, multiplied by $62\frac{1}{2}$ lb., (the weight of a cubic foot of water,) gives the number of pounds avoir. the wheel can sustain, acting perpendicularly at its circumference without

any motion. If this number be lessened by any quantity, the wheel will move.

It is evident that the effective velocity of the water against the wheel is the relative velocity between the water and the wheel, or the difference of their velocities. But suppose (for the present) that a mill works with the greatest effect when the velocity of the wheel is one-third that of the water; since therefore the power to impel the wheel is in proportion to the square of the relative velocity of the water and wheel, we must multiply the above calculated number by $\frac{4}{9}$, ($\frac{2}{3}^2$ = the square of the difference of the velocities,) and again by one-third, in order that the wheel may move with one-third the velocity of the water, and the product is evidently the number of pounds the wheel can raise vertically, with a velocity equal to one-third that of the water in feet per second.

According to Messrs Watt and Bolton, a horse is capable of raising 32000 pounds 1 foot high in a minute. Wherefore,

To calculate the number of horse-powers.

Multiply the number of pounds raised by the waterfall, by 60 times the number of feet they are raised per second; and divide the product by 32000, the quotient is the number of horse-powers of the fall.

EXAMPLE.

A certain brook runs at the rate of four feet per

second, the area of its cross section in that place is six feet, and a fall can be obtained of 16 feet; required the power of the fall for an undershot wheel.

$4 \times 6 = 24 =$ the number of cubic feet flowing per second.

$\sqrt{16 \times 64.38} = 32+ =$ the velocity of the water at the end of the fall.

$24 \div 32 = \frac{3}{4}$ feet $=$ the section of the stream at the end of the fall.

$\frac{3}{4} \times 16 \times 62\frac{1}{2} = 750$ pounds $=$ the weight sustained without motion.

$750 \times \frac{1}{3} \times \frac{4}{9}$ (the square of the relative velocity,) $= 111\frac{1}{9}$ pounds raised through $10\frac{2}{3}$ feet per second, (velocity of the wheel.)

$111\frac{1}{9} \times 10\frac{2}{3} \times 60 \div 32000 = 2\frac{2}{9} =$ number of horse-powers.

A reason will be given next page, that this number should be multiplied by $2\frac{1}{4}$, viz. $2\frac{2}{9} \times 2\frac{1}{4} = 5$ horse-powers.

PROPOSITION LXXVI.

An undershot wheel performs most work when the velocity of the wheel is one-third that of the water, providing the water escapes from the wheel immediately after its impact.

Demonstration.

Let $v =$ the velocity of the water.

$x =$ the velocity of the wheel.

Then $v - x =$ the relative velocity.

Consequently, $\overline{v^2-2vx+x^2}\times x = x^3-2vx^2+v^2x$ a maximum; and its fluxion, $3x^2\dot{x}-4vx\dot{x}+v^2\dot{x}=0$, or $x^2-\frac{4}{3}vx+\frac{4}{9}v^2=\frac{4}{9}v^2-\frac{1}{3}v^2$, or $x=\frac{2}{3}v\pm\sqrt{\frac{4}{9}v^2-\frac{1}{3}v^2}$ $=\frac{2}{3}\ v\pm\sqrt{\frac{1}{9}v^2}=\frac{2}{3}v\pm\frac{1}{3}v=\frac{1}{3}v$ a maximum, or v a minimum.

PROPOSITION LXXVII.

An undershot wheel is capable of raising only $\frac{4}{27}$ of the water expended to the height of the fall.

Demonstration.

Let $v=$ the velocity of the stream.
$Q=$ the quantity of water expended per second.
$H=$ the height of the fall.

Then $QH=$ the momentum of the fall; and $\frac{Q}{v}\times H\times\frac{4}{9}\times\frac{1}{3}v=$ the momentum of the wheel; that is, $QH : \frac{4}{27}\frac{QHv}{v}$: : momentum of the water : momentum of the wheel, or $27:4$: : momentum of the water : momentum of the wheel.

Mr Smeaton found, by his numerous experiments on mills, that the greatest effect was produced in an undershot wheel, when its velocity was between $\frac{1}{3}$, and $\frac{1}{2}$ that of the water; being nearer to $\frac{1}{2}$ than $\frac{1}{3}$. The reason of this variation from the theory is, that the water being confined into a narrow channel does not escape immediately after it impinges upon the floats, (as the theory supposes,)

but is heaped up upon the floats to about $2\frac{1}{2}$ times its natural depth, and acts partly by its weight, as well as by its force, in a manner not easily subjected to strict investigation.

He also discovered, that for the same reason a well-constructed undershot wheel is capable of raising $\frac{1}{3}$, instead of $\frac{4}{27}$, of the water expended, being $2\frac{1}{4}$ times more than that given by theory.

That the effective head being the same, the effect will be nearly in proportion to the quantity of water expended.

The expense of water being the same, the effect will be nearly in proportion to the height of the effective head.

The quantity of water being the same, the effect is nearly in proportion to the square of the velocity.

The aperture being the same, the effect is nearly in proportion to the cube of the velocity.

None of the above discrepancies can obtain, in the application of the theory, to the paddle wheels of steam vessels, because the water is allowed to escape in all directions. The section of the water acting against the paddles is equal in all velocities, and the force to impel the vessel is in proportion to the square of the velocity of the paddles. (Prop. LXXV. and Cor. 2.)

But because the resistance the vessel meets in the water is in proportion to the square of her velocity, the velocity of the vessel is therefore in proportion to the velocity of the paddles simply.

The velocity of the paddles against the water is

evidently the relative velocity, or the difference of their velocity and that of the vessel, or when the vessel is backed their sum; but since the difference of the velocities increases or decreases uniformly with the velocities, the velocity of the vessel is still in proportion to the absolute velocity of her paddles.

In similar circumstances, therefore, the way of a steam-ship can be measured from the number of strokes of her engine per minute.

PROPOSITION LXXVIII.

If the water in the buckets of an overshot wheel be supposed to be equally diffused over the half circumference of the wheel, then the whole weight of the water in the bucket : is to its force to turn the wheel : : as the half circumference of a circle : is to its diameter.

Demonstration. Because the force required to sustain the water upon any small portion of the circumference, considered as an inclined plane, AB : is : to its whole weight : : as A*b* : to AB. (Fig. 99.) The force that sustains the water upon BC : to its whole weight : : as BD or *bc* : to BC, and so on with all the other portions: That is, as the sum of all the forces sustaining the water upon the different portions of the wheel : is to its whole weight : : so is the sum of all the perpendiculars or the diameter : to the sum of all the portions or the semi-circumference.

PROPOSITION LXXIX.

An overshot wheel will raise nearly as much water to the height of the fall as is expended in impelling it; the weight of the fall being reckoned from the bucket that receives the water to the bucket that discharges it.

Demonstration. For let the opposite side of the wheel be equally and similarly loaded, the wheel, according to the principles of the lever, will rest in any position; and a weight sufficient to overcome the friction laid on one side will cause the other to ascend.

Overshot mills act chiefly by the weight of water in the buckets of the wheel, and their greatest performance must evidently be when the product of the weight of water in the buckets by the velocity of the wheel is greatest. On a superficial view of this subject, it should seem, that, since the quantity of water in the buckets is inversely as the velocity, the product of the wheel's velocity, by the weight of water, must be equal in all velocities of the wheel. But there is another circumstance to be considered, namely, the water cannot expend its weight upon the wheel until it fall through a height to acquire a velocity equal to that of the wheel; for it cannot overtake the buckets till then. When the wheel moves rapidly, therefore, all the weight of the water is lost until it reach the buckets.

According to Mr Smeaton, the velocity of an

overshot wheel should be between 2 and 4 feet per second for all sizes.

PROPOSITION LXXX.

A breast-mill is impelled partly by the velocity of the water, and partly by its weight. Its power is that of an overshot wheel, whose height is the perpendicular space between the receiving and discharging buckets, added to that of an undershot, whose height is the perpendicular space between the head of the fall and the receiving bucket. A breast-mill is therefore the more efficient the nearer it approaches in its construction to an overshot.

An undershot wheel is the best for a large supply of water with a small fall. An overshot for a high fall with little water; and a breast-wheel when both are moderate.

THE CONSTRUCTION OF MILL-WRIGHTS' TABLES.

1. By levelling and measuring, find the height of the fall of water; its height being estimated from its upper surface to the middle of the depth of the stream where it acts upon the float-boards.

2. Find the velocity acquired by the water in falling through that height, by multiplying the height of the fall by 64.38, and extract the square root; then substract $\frac{1}{10}$ of the root for the friction of the water; the remainder is the velocity.

3. Find the velocity that ought to be given to the float-boards, by taking $\frac{5}{7}$ of the velocity of the water, that gives the number of feet they move through in 1 second to produce the greatest effect.

4. Divide the circumference of the wheel by the velocity of its float-boards in feet per second; the quotient is the number of seconds the wheel takes to make one revolution.

5. Divide 60 by this last number; the quotient is the number of revolutions of the wheel in 1 minute.

6. Divide 90 by the number of turns of the wheel in 1 minute; the quotient is the number of turns of the millstone for one of the wheel; 90 being the number of turns that a millstone of 5 feet diameter ought to have in a minute.

7. Then as the number of turns of the wheel in a minute : is to the number of turns of the millstone per minute : : so is the number of staves in the trundle : to the number of teeth in the spur-wheel, in the nearest whole numbers.

8. Multiply the number of turns of the wheel in a minute, by the number of turns of the millstone for 1 of the wheel; the product will be the number of turns of the millstone. Or, after having

found the nearest whole number for the teeth of spur-wheel, multiply that number by the number of revolutions of the water-wheel per minute, and divide the product by the number of staves in the trundle; the quotient is the true number of turns of the millstone per minute.

In this manner the following Table has been calculated for a water-wheel 15 feet in diameter, the millstone being 5 feet diameter, and turning 90 times in a minute.

In the following Table the velocity of the water is calculated upon the supposition, that a body falls 16.087 feet in a second; but, according to the recent and very accurate experiments of Captain Kater, the number should be 16.095. The difference, however, is so small, that no error can arise from it in practice.

A MILLWRIGHT'S TABLE,

In which the Velocity of the Wheel is Three-Sevenths of the Velocity of the Water, allowance being made for the Effects of Friction on the Velocity of the Stream for a Wheel Fifteen Feet Diameter.

Height of the Fall of Water.	Velocity of the Water per Second.	Velocity of Wheel per Second, being 3-7th of that of the Water.	Revolutions of the Wheel per Minute.	Number of Revolutions of the Mill-stone for 1 of the Wheel.	Teeth in the Wheel, and Staves in the Trundle.		Revolutions of the Mill-stone per Minute by these Staves and Teeth.
Feet.	Feet. 100 Parts of a Foot.	Feet. 100 Parts of a Foot.	Revolutions. 100 Parts of a Revol.	Revol. 100 Parts of a Revol.	Teeth.	Staves.	Revol. 100 Parts of a Revol.
1	7.62	3.27	4.16	21.63	130	6	90.07
2	10.77	4.62	5.88	15.31	92	6	90.16
3	13.20	5.66	7.20	12.50	100	8	90.00
4	15.24	6.53	8.32	10.81	97	9	89.67
5	17.04	7.30	9.28	9.70	97	10	90.02
6	18.67	8.00	10.19	8.83	97	11	89.86
7	20.15	8.64	10.99	8.19	90	11	89.92
8	21.56	9.24	11.76	7.65	84	11	89.80
9	22.86	9.80	12.47	7.22	72	10	89.68
10	24.10	10.33	13.15	6.84	82	12	89.86
11	25.27	10.83	13.79	6.53	85	13	90.16
12	26.40	11.31	14.40	6.25	75	12	90.00
13	27.47	11.77	14.99	6.00	72	12	89.94
14	28.51	12.22	15.56	5.78	75	13	89.77
15	29.52	12.65	16.13	5.58	67	12	90.06
16	30.48	13.06	16.63	5.41	65	12	90.06
17	31.42	13.46	17.14	5.25	63	12	89.99
18	32.33	13.86	17.65	5.10	61	12	89.72
19	33.22	14.24	18.13	4.96	60	12	90.65
20	34.17	14.64	18.64	4.83	58	12	90.09

A NEW AND EASY RULE

For finding the power of a fall of water.

1. For an undershot. Multiply the height of the fall by the quantity of water flowing per minute both in feet, and divide the product by 5000; the quotient is the number of horse-powers.

2. For an overshot. Find the power for an undershot, and multiply it by 2½; the product is the horse-powers.

3. For a breast-wheel. Find the power for an undershot from the height of the fall to where the water enters the bucket. Then for an overshot for the rest of the fall, add the two together; the sum is the power of the breast-wheel.

The above rules make allowance for the friction and waste of water.

Investigation of the Rules.

Because the power of a fall is as the height and quantity of water conjointly, according to Smeaton's maxims, as also by theory. And in the last example we find, that a fall of 16 feet, with 24 cubic feet of water in a second, or 1440 feet in a minute, gives 5 horse-powers. Therefore $5 : 1440 \times 16 = 23040 :: 1 : 4601$, and making allowance

for friction and waste, I call this last number 5000 for the divisor.

EXAMPLE.

What is the power of a fall of water flowing with a velocity of 20 feet per minute, its section in that place measuring 10 feet, and a fall can be obtained of 30 feet.

$20 \times 10 \times 30 = 6000$

$6000 \div 5000 = 1.2$ horse-powers for an undershot.

$1.2 \times 2.5 = 3$ ———— for an overshot.

As water-power is used for many kinds of machinery, I thought it most proper to give it in horse-powers. It will require about 3 horse-powers of any kind to grind 1 imperial quarter of wheat in 1 hour, which is near enough for practice.

For a farther account of mills, see Mr Smeaton's Treatise on Mills, and Dr Brewster's Appendix to Ferguson's Lectures.

PROPOSITION LXXXI.

If two fluids, A and B, (fig. 106, 107, 108,) sustain one another in two separate vessels that communicate with one another, their heights above their place of meeting C, will be inversely as the densities of the fluids.

Demonstration. For in order that they sustain

one another, it is necessary that the columns of the fluids, whose common base is C, should be of the same weight, which cannot be unless the height of the column be respectively inversely as their densities.

PROPOSITION LXXXII.

Air, or other gaseous bodies, (besides having the properties of fluids in general,) have mechanical properties peculiar to themselves. They are perfectly elastic or compressible. Any quantity of air is capable of being compressed into almost the smallest assignable space without impairing its elasticity; also, if all pressure were withdrawn from it, the smallest quantity is capable of expanding itself into a very great space. Experiments show, that its density and also its elasticity are directly as the pressure applied to it. The elasticity of air for small compressions is as the densities; but when the compressions are great, the elasticity is in a less ratio than the densities. Thus, when air is compressed into the $\frac{1}{8}$th of its original dimensions, its elasticity is 6.835, according to Sulzer.

Air has but a small degree of weight; but the weight of the atmosphere or the whole mass of air surrounding our globe is immense; for experiments prove, that its weight at the surface of the earth is $14\frac{1}{2}$ pounds avoir. upon every square inch of surface at a medium, or equal to a column of mercury of the same base, and $29\frac{1}{2}$ inches high;

so that the weight of air, supported by an ordinary-sized man, whose surface may be 14 square feet, is nearly 30,000 pounds; which weight would be insupportable were it not that it is counterbalanced by an equal pressure outward. It may be shown, that the number of superficial feet upon the surface of the earth is about 5576000000000000, which, multiplied by 2088, gives 11642688000000000000 pounds for the whole weight of the atmosphere.

Take a strong straight glass tube AB, about 3 feet long, turned up at the end B. Let the end B be open, and that at A close. Fill the tube perfectly with mercury, by holding it in a position nearly horizontal, and shaking it to discharge all the air; then turn up the close end A, and hold the tube vertically, the mercury will descend to C, and will be balanced by the pressure of the atmosphere upon the surface of the mercury at B. (Prop. LXXXI.) Whatever the area of the surface at B may be, a column of air to the top of the atmosphere is equal to the weight of the column BC of mercury of the same base. The portion AC of the tube contains a perfect vacuum.

This instrument, when inserted into a case, with a scale attached to it, is the barometer: it indicates the weight and variations of the atmosphere at all times; by it also the measurement of the heights of mountains, &c. may be approximated to; because the pressure of the atmosphere decreases in geometrical progression with the height: it being

determined by experiment that it is 4 times rarer for every 7 miles of ascent.

For the method of measuring heights with the barometer, see Brown's Logarithms.

PROPOSITION LXXXIII.

Since the weight of mercury to that of water is as 13.57 to 1, it requires a column of water 33 feet 4 inches high to balance a column of air of the same base at a medium; and consequently a common pump, if perfectly air-tight, would lift a column of water under the piston of that height, or rather the weight of the atmosphere upon the surface of the water in the open well, would force the water to that height in the pump, if all the air was completely drawn off by the piston. But as no pump can be made so perfect as to exclude all the air, great allowance must be made. A pump may lift water from any depth, if the piston be low enough; but if it be little more than 20 feet above the surface of the water it will not work.

PROPOSITION LXXXIV.

The resistance that a sphere encounters in moving through a fluid is one-half of that of the plane of a circle of the same diameter. (Fig. 109.)

Demonstration. Let ABC be a sphere moving in direction CD, AB a plane passing through its centre perpendicular to the line of direction CD,

and let CDFE be an indefinitely small column of the fluid acting upon the surface CE. Produce this column to GH, it is evident that the small surface CE : surface GH :: line CE : line GH :: radius : sine of the angle of incidence :: resistance on GH : resistance on CE, (Prop. XLI.) Therefore the resistance of all the small parts GH, or the whole surface of the plane AB : to all the small portions CE, or the whole surface of the hemisphere ACB :: 2 : 1, for the surface of the inscribed circle is $\frac{1}{4}$ of the surface of the sphere.

Cor. 1. The resistance of a cylinder moving in a fluid sideways, from the same mode of reasoning, may be shown to be to that of a plane of the same length and breadth as the diameter of a circle to half its circumference.

Cor. 2. Whence we see the propriety of having tall slender vents and other such structures circular, as being the best form for lessening the resistance of the wind.

PROPOSITION LXXXV.

The resistance that similar bodies encounter in moving through fluids is as the density of the fluid, the square of the diameter of the bodies, and the squares of the velocities directly.

This follows from Prop. LXXIV. and Corollaries.

Cor. The motion destroyed by the resistance is as the densities of the bodies and the cubes of the diameters inversely.

Demonstration. Because the quantities of matter are as the densities, and the cubes of the diameters of the bodies directly; and the velocity generated or destroyed by the same force is inversely as the quantity of matter or weight of the body moved.

Cor. 2. The effect of resistance upon small bodies is greater than that upon large bodies, because the force of the resistance is in proportion to the surface, or the square of the diameter; but the force to oppose that resistance is in proportion to the solidity, or the cube of the diameter. From this we infer, that small bodies in falling from a height, sooner attain the maximum of their velocity or resistance whether in air or water; that is, they sooner attain a resistance equal to their own weight than larger bodies, which is one of the reasons why they are less damaged by a fall than large bodies. Also small shot does not carry so far as large; nor musket-balls so far as those of a cannon, even though they were of the same metal, and discharged with the same velocity. Also it is much easier for insects to fly than birds; and the larger the fowl is the greater is the effort in flying. Nature seems, therefore, to have set very limited

bounds to the weight of flying animals, the largest eagles being only 30 pounds. It would require for an animal of 150 pounds weight, to keep itself merely afloat in the air, an incessant resistance made against it equal to that weight, or to move wings of 227 feet in area, with an incessant velocity of 17 feet per second against the air downward, without any resistance upward, merely to keep itself from falling. Whence the extreme difficulty, if not impossibility, of men to fly in the air by any application of their own strength alone to any machinery, as may be calculated from the following Table:—

TABLE

Showing the Pressure of the Wind for the following Velocities, from the Philosophical Transactions of the Royal Society of London.

Velocity of the Wind.		Force upon 1 Square Foot in Pounds Avoir.
Miles in 1 Hour.	Feet in 1 Second.	
1	1.47	.005
2	2.93	.020
3	0.40	.044
4	5.87	.079
5	7.33	.123
10	14.67	.492
15	22.00	1.107
20	29.34	1.968
25	36.67	3.075
30	44.01	4.429
35	51.34	6.027
40	58.68	7.873
45	66.01	9.963
50	73.35	12.300
60	88.02	17.715
80	117.36	31.490
100	146.70	49.200

In the preceding Table the force agrees with the square of the velocity, according to the theory. But Dr Hutton, of the Royal Academy, Woolwich, found, by experiments, that the force augmented in a ratio rather greater than the square of the velocity, namely, in the 2.04 power. He also discovered the resistances to several forms of bodies to be as under:—

Velocity per Second in Feet.	Hemisphere.		Cone.		Cylinder.	Whole Globe.
	Flat Side.	Round Side.	Vertex.	Base.		
3	.051	.021	.028	.064	.050	.027
5	.148	.063	.071	.162	.143	.068
10	.573	.242	.260	.587	.576	.255
15	1.336	.552	.589	1.346	1.327	.581
20	2.542	1.033	1.069	2.540	2.528	1.057

The greatest diameters of the above bodies was $6\frac{3}{8}$ inches, and the areas of the flat sides were therefore $\frac{2}{9}$ of a foot square; the lengths of the cone and cylinder were the same as the breadth.

Dr Hutton found also, that when the velocity was very great, the resistance increased gradually in the ratio to the velocity, till at the velocity of 2000 feet in a second it was as the 2.1 power of the velocity.

CALCULATION OF BALLOONS.

From the consideration, that a body ascends in a fluid of greater specific gravity than itself, it is

easy to calculate the size of a balloon that will ascend with any given weight, and inflated with gas, whose specific gravity is known. For instance, let it be required what size of a spherical balloon, inflated with hydrogen gas, will ascend with 400 pounds avoir., including its own weight.

The specific gravity of hydrogen gas is 0.1, and of atmospheric air 1.2. A cubic foot of hydrogen gas is therefore 0.1 oz. and a cubic foot of atmospheric air 1.2 oz., and their difference is 1.1 oz. Consequently as 1.1 oz. : 400 lb., or 6400 oz. :: 1 foot : to 5818 feet, which is the quantity of gas that will sustain 400 lb., which (by mensuration) is the solidity of a sphere 22.3 feet diameter, and any thing greater will ascend with 400 lb.

It will ascend, however, only to such a height, where the weight will be equal to that of an equal bulk of the more rarefied air, where it will remain stationary; but if weight be thrown out, it will again ascend.

A balloon in the higher regions of the atmosphere, if too much inflated, is in danger of bursting from a want of pressure upon its external surface; the rarity of the air making it insufficient to counterbalance the elasticity and expansion of the gas in the balloon. Whence the necessity of a valve to allow the gas to escape when occasion requires.

CALCULATION OF PARACHUTES.

When the resistance encountered by a body falling through air, or other fluid, becomes equal to the force of gravity, or the weight of the body, its motion will then be no longer accelerated, but it will descend with a uniform motion. From this evident circumstance, the resistance, the velocities, and the dimensions of parachutes can be calculated.

For example. What must the diameter of a parachute be, that will allow a weight of 150 lb., including its own weight, to descend with the velocity a body would acquire by falling 6 feet in free space?

A body acquires a velocity of 19.65 feet per second in falling through 6 feet, (Prop. XV. and XVI.); and by the above table of resistances, as 4.4^2 feet : 19.65^2 feet : : .044 lb. : .877 lb. = the force upon 1 foot, and as .877 lb. : 150 lb. : : 1 foot : 171 feet = the area of the parachute, or a circle 14 feet 9 inches diameter.

MISCELLANEOUS QUESTIONS.

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1. If a racehorse, with his rider, weighing 2240 lb., in 3 seconds after starting acquire the velocity of 15 feet in a second, what is the force exerted by his limbs?

$32.19 \times 3 = 96.57$ = the velocity acquired by a body falling in 3 seconds. But the force is as the velocity in equal times. Therefore 96.57 feet : 15 feet : : 2240 lb. : 350 lb., which would be the true force if expended upon nothing but his progressive motion. But if in that time he acquires his greatest velocity, he would move on without farther effort, if there was no resistance, (Axiom I;) his whole force afterwards is therefore employed in keeping up the angular motion of his limbs and overcoming other resistances; and we may reasonably suppose, that one-half his force is employed in overcoming such obstacles from the time of starting. Wherefore

$350 \times 2 = 700$ pounds = the force exerted horizontally.

2.* A flat rope is coiled upon itself round a cylinder, driven once round for every stroke of an engine, for the purpose of drawing coals from a pit, while at the same time an empty basket is let down by another flat rope coiled round an equal cylinder upon the same axle. The depth of the pit is $= d$, ($= 100$ fathoms or 7200 inches,) the diameter of cylinder $= a$, ($= 6$ feet or 72 inches,) and the edge of the rope $= t$, ($= 1$ inch); required the number of strokes of the engine or turns of the cylinder ($= n$) for each draught.

Let 2DC, (fig. 92,) = diameter of cylinder $= a$, and $.7854 = c$, dt is the area of the edge of rope or ring AD, and $a^2c =$ area of cylinder. But $(a+2^2nt)^2c - a^2c$ is also the area of the ring AD; wherefore $n^2+\frac{an}{t}=\frac{d}{4ct}$, and by the rules of quadratics $n = \sqrt{\left(\frac{d}{4ct}+\frac{a^2}{4t^2}\right)} - \frac{a}{2t}$.

Rule in words. Divide the depth of the pit by 3.1416 times the thickness of rope; to the quotient add the square of the diameter of cylinder, divided by four times the square of the thickness of rope; from the square root of the sum take the diameter of cylinder divided by twice the thickness of rope, the remainder is the number of strokes of the engine.

* Questions 2 and 3 are given and solved by Mr Robert Moor, engineer, Birsley coal-work.

$$\sqrt{\frac{7200}{3.1416\times1}+\frac{72^2}{4\times1^2}}-\frac{72}{2\times1}=\sqrt{2290+1296}-36$$
$= 24$ nearly.

Since $n^2+\frac{an}{t}=\frac{d}{4ct}$, then $a=\frac{d}{4nc}-nt$, $t=\frac{d}{4n^2c}-\frac{a}{n}$, and $d=(a+nt)4nc$.

3. To find how far from the bottom (m) the baskets will meet, having the number of strokes, the thickness of rope, and diameter of cylinder given.

They will evidently meet when the coils of rope upon each cylinder are of equal thickness, or at half the number of strokes; therefore $(a+nt)^2.-a^2=m$, or $(2a+nt)ntc=m$.

In words. To twice the diameter of cylinder add the number of strokes multiplied by the thickness of rope, multiply this sum by the product of .7854 times the number of strokes multiplied by the thickness of rope; the last product is the distance from the bottom where the baskets will meet.

Thus $(\overline{72\times2}+\overline{24\times1})\times24\times1\times.7854=4032\times.7854=3167$ inches, or 44 fathoms nearly, or 56 fathoms from top.

Having the weight that will crush a given body, and the weight that will draw it asunder length-ways; to find the weight that will break it across.

Let EBCF (fig. 83,) be a beam of any material to be broken across at the section EF, it is evident

PLATE VII.

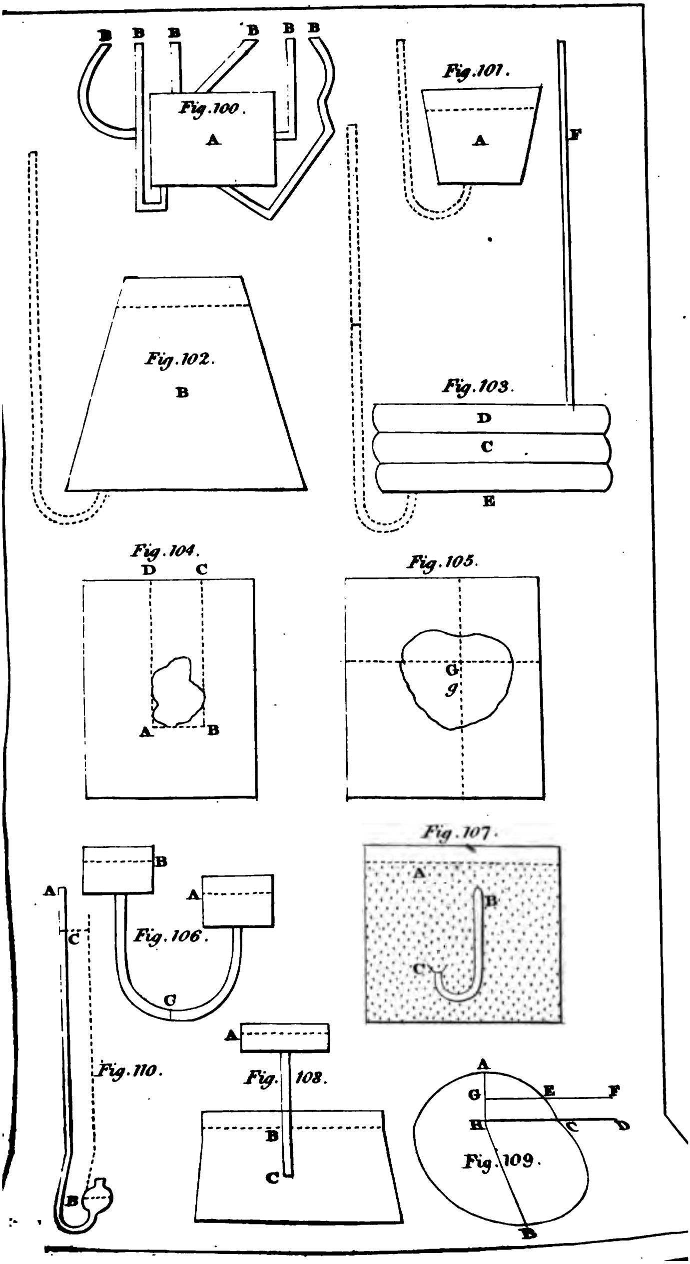

Fig. 100.
Fig. 101.
Fig. 102.
Fig. 103.
Fig. 104.
Fig. 105.
Fig. 106.
Fig. 107.
Fig. 108.
Fig. 109.
Fig. 110.

there must be a neutral point or fulcrum somewhere in the section EF, round which the body will turn in breaking; that all the fibres or particles in the part of the section above that point will be torn asunder, while those below it will be crushed inward; that this neutral point will accommodate itself to a situation higher or lower, according as the strength opposed to tearing or crushing may predominate, or to such a point that the forces will balance one another, and that the force exerted upon either section will be equal to the square of its depth in inches, multiplied by the force of 1 inch.

Let $\left.\begin{array}{l} L = \text{the length of the beam} \\ D = \text{its depth} \\ B = \text{its breadth} \end{array}\right\}$ in inches.

t = the cohesive force of 1 inch.

c = the strength opposed to crushing 1 inch.

x = the part of the section above the neutral point.

y = the part of the section below that point.

F = the whole force required to break the beam across.

Then $tx^2 = cy^2$, $x+y = D$, and $(tx^2+cy^2)\frac{B}{L} = F$.

Consequently $x = y\sqrt{\frac{c}{t}} = D - y$, or $y = \frac{D}{\sqrt{\frac{c}{t}}+1}$, and

$x = \dfrac{D\sqrt{\frac{c}{t}}}{\sqrt{\frac{c}{t}}+1}$. These values of x and y being substituted in the third equation, we obtain $F = \dfrac{(2ct)D^2B}{(t+c+2\sqrt{ct})L}$. Or if D and B be made each $= 1$, $\dfrac{2ct}{(t+c+2\sqrt{ct})L} = F$.

Rule in words. Take twice the product of the cohesive and resistive forces of one inch for a dividend. Add the sum of the two forces simply to twice the square root of their product, and multiply the sum by the length of the beam in inches for a divisor. Divide the dividend by the divisor; the quotient is the weight required to break the beam across of 1 inch square when supported at one end. For any other dimensions, multiply by the breadth and square of the depth.*

EXAMPLES.

4. The cohesive force of Craigleith stone is 800 lb. for 1 inch, and is crushed with 8000 lb. in round numbers. What weight will break a lintel across 48 inches long, 16 deep, and 9 broad?

$$\frac{8000\times800\times2}{48\times(8000+800+2\sqrt{(8000\times800)})} = \frac{12800000}{(8800+5059)\times48} = 19.24$$

lb. = the weight that breaks 1 square inch supported at one end; therefore,

* This important theorem, the discovery of the Author, he submits to the attention of mathematicians and engineers in general.

$19.24 \times 16^2 \times 9 \times 2 = 88658$ lb. $= 39\frac{1}{2}$ tons $=$ the weight that will break the lintel across supported at both ends, the weight in its middle.

5. The cohesive strength of oak is 11880 lb. on an inch, and is crushed with 3860 lb.; required the weight that will break a beam across 8 feet long, and 4 inches square, supported at both ends.

$$\left(\frac{11880 \times 3860 \times 2}{11880+3860+2\sqrt{11880 \times 3860}}\right) \times \frac{4^2 \times 4}{96} = \frac{91713600 \times 64}{29283 \times 96} =$$

2088 lb. $=$ the weight that will break it supported at one end; therefore $2088 \times 2 = 4176$ lb. $=$ the weight that will break it supported at both ends, which also agrees very nearly with experiments.

6. The cohesive strength of cast-iron is 19000 lb. for 1 inch, and it is crushed with 160000, (Mr Rennie's experiments in round numbers,) what weight will break 1 foot long, and 1 inch square across, being supported at both ends?

$$\frac{160000 \times 19000 \times 2}{(160000+19000+2\sqrt{(160000 \times 19000)} \times 12} = \frac{6080000000}{3471264} =$$

1750 lb. $=$ the weight that will break it if supported at one end; therefore $1750 \times 2 = 3500$ lb. the answer, agreeing also with experiments.

7. A bar of cast-iron 4 feet long, and $\frac{1}{2}$ inch square, supported at one end, can just bear 43 lb.; required the dimensions of a bar of the same metal,

similarly situated, and every way proportional to the former, that can just bear its own weight and no more?

The weight of the given bar is 3.13 lb., according to the tables; therefore (Prop. LXIII.) $4+\frac{2\times4\times43}{3.13}=4+110=114$ feet long; and 4 feet : 114 feet :: $\frac{1}{2}$ inch : $14\frac{1}{4}$ inches square.

THE END.

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ERRATA.

Page 7, line 5 from the bottom, for *at* read *in*.

—— 26, —— 15, for $\overline{39.1386T^2}$ read $39.1386T^2$.

—— 31, —— 3 from the bottom, erase the word *equal*.

DIRECTIONS TO THE BINDER FOR PLACING THE PLATES.

Plate	I. to face	page 18
——	II.	36
——	III.	58
——	IV.	70
——	V.	104
——	VI.	142
——	VII.	190

The plates to open to the right hand, beyond the letter-press.